

Introduction to limits of functions

Tuesday, September 9, 2025 3:00 PM

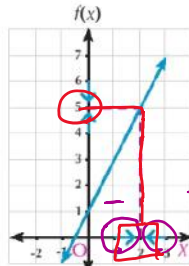
Lim Lim

Limits of a function at a point

Study the values of the function f where $f(x) = 2x + 1$ when x close to 2 from the next data:

$x > 2$	$f(x)$
2.1	5.2
2.01	5.02
2.001	5.002
2.0001	5.0002
...	...
$x \rightarrow 2^+$	$f(x) \rightarrow 5$

$x < 2$	$f(x)$
1.9	4.8
1.99	4.98
1.999	4.998
1.9999	4.9998
...	...
$x \rightarrow 2^-$	$f(x) \rightarrow 5$



Handwritten notes and diagrams:

- A large blue circle around the number 5.
- A diagram showing a vertical line at $x=2$ with arrows pointing left and right towards it, labeled $-\infty$ and $+\infty$ respectively.
- Two boxes containing $x \rightarrow 2^-$ and $x \rightarrow 2^+$.
- The equation $\lim_{x \rightarrow 2} (2x + 1) = 5$ written in purple.

We see that:

when x approaches to 2 from the right and from the left then, $f(x)$ approaches to 5. We express that mathematically as $\lim_{x \rightarrow 2} (2x + 1) = 5$ and the graphical representation of f illustrating that.

Definition:

Definition If the value of the function $f(x)$ approaches to the real number l when x close to the real number a , then $\lim_{x \rightarrow a} f(x) = l$

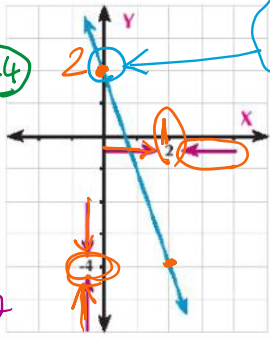
Example

Estimate $\lim_{x \rightarrow 2} (2 - 3x)$ graphically and numerically.

Handwritten calculations:

$$y = 2 - 3x = 2 - 3 \times 2 = 2 - 6 = -4$$

$$\lim_{x \rightarrow 2} (2 - 3x) = -4$$



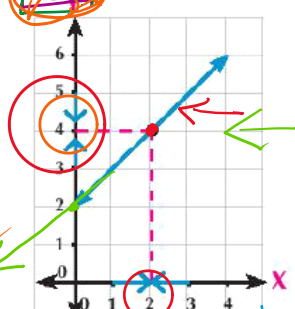
x	1.99	1.999	2	2.01	2.1
f(x)	-3.97	-3.997	-4	-4.03	-4.3

Estimate graphically and algebraically $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

Handwritten algebraic simplification:

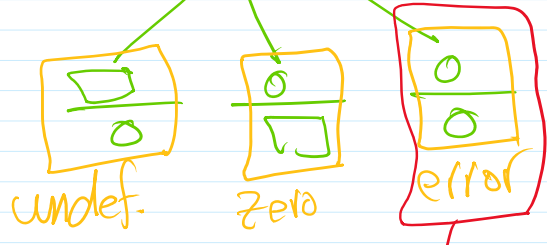
$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \frac{4 - 4}{2 - 2} = \frac{0}{0}$$

Handwritten note: "error" with a red circle around it.



Handwritten algebraic simplification:

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$



Handwritten note: $\frac{0}{2} = \text{zero}$

Handwritten note: $\frac{2}{0} = \text{undefined}$

Handwritten algebraic simplification:

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 4}}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2}$$

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 - 4}}{x - 2} = \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} \leftarrow$$

$$\lim_{x \rightarrow 2} (x+2) = 2+2 = 4 \leftarrow$$

$$f(x) = \left(\frac{x^3 - 1}{x - 1}\right) - 2 \text{ when } x \rightarrow 1$$

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60%
factorize

$$\frac{x^2 - 4}{x - 2} = y$$

$$y = \frac{-4}{2} = -2$$

Unit 1 term 2 grade
Lesson 1, 2, 3, 4, 5

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$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \frac{1 - 1}{1 - 1} = \frac{0}{0} \text{ Problem}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^2 - 2x + 1)}{x - 1}$$

factorize

grades unit 1 lessons 1, 2, 3, 4, 5

$$\lim_{x \rightarrow 1} (x^2 - 2x + 1) = 1 - 2 + 1 = 0$$

$$\neq 2 \times 1 + 1$$

2nd

x	0.9	0.99	0.999	1	1.001	1.01	1.1
y	0	0	0	0	0	0	0

