

# Finding the limit of a function algebraically

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**Theorem**

> If  $f(x)$  is a polynomial,  $a \in \mathbf{R}$

Then:  $\lim_{x \rightarrow a} f(x) = f(a)$

**Direct Substitution**

1 Find each of the following limits:

a  $\lim_{x \rightarrow 2} (x^2 - 3x + 5) = 2^2 - 3 \times 2 + 5 = 4 - 6 + 5 = 3$

b  $\lim_{x \rightarrow 3} (-4) = -4$

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**Theorem**

> If  $\lim_{x \rightarrow a} f(x) = l$ ,  $\lim_{x \rightarrow a} g(x) = m$  then:

1-  $\lim_{x \rightarrow a} k f(x) = k.l$  where  $k \in \mathbf{R}$

2-  $\lim_{x \rightarrow a} [f(x) \pm g(x)] = l \pm m$

3-  $\lim_{x \rightarrow a} f(x) \cdot g(x) = l.m$

4-  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{l}{m}$  where  $m \neq 0$

5-  $\lim_{x \rightarrow a} (f(x))^n = l^n$  where  $l^n \in \mathbf{R}$

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$\lim [g(x) + f(x)] = \lim g(x) + \lim f(x) = m + l$

$\lim f(x) \cdot \lim g(x) = l \cdot m$

$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}$

2 Find the following limits:

a  $\lim_{x \rightarrow -1} \frac{3x+7}{x^2+2x-5}$

b  $\lim_{x \rightarrow -2} (\sqrt{4x^2-3})$

*Handwritten work for (a):*

$$\lim_{x \rightarrow -1} \frac{3x+7}{x^2+2x-5} = \frac{3(-1)+7}{(-1)^2+2(-1)-5} = \frac{-3+7}{1-2-5} = \frac{4}{-6} = -\frac{2}{3}$$

*Handwritten work for (b):*

$$\lim_{x \rightarrow -2} (\sqrt{4x^2-3}) = \sqrt{4(-2)^2-3} = \sqrt{16-3} = \sqrt{13}$$

2 Use Factorization: (1) Direct sub.

$\lim_{x \rightarrow a} [f(x)]^n$

$\lim_{x \rightarrow a} f(x) = l \Rightarrow \lim_{x \rightarrow a} f(x)^n = l^n$

*Handwritten example:*

$$\lim_{x \rightarrow \frac{1}{5}} (3x)^2 = \left( \lim_{x \rightarrow \frac{1}{5}} 3x \right)^2 = \left( 3 \cdot \frac{1}{5} \right)^2 = \left( \frac{3}{5} \right)^2 = \frac{9}{25}$$

$= \frac{-6}{-6} = \boxed{3}$        $-10 \rightarrow = (10) \quad - \text{VIS}$        $X^{\frac{3}{5}} = (X^{\frac{3}{5}}) = \sqrt[5]{X^3}$

② Use Factorization: ① Direct sub.

③ Use factorization to find the following limits:

a)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \frac{1-1}{1-1} = \frac{0}{0}$

b)  $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 1}{x^2 + x - 2}$

a)  $\lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)}$

$= \lim_{x \rightarrow 1} x^2 + x + 1 = \boxed{3}$

$\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 1}{(x-1)(x+2)}$

Long division

Long division process for  $\frac{x^3 - 2x^2 + 1}{x - 1}$ :

$x^3 - 2x^2 + 1$  divided by  $x - 1$ .

Step 1:  $x^3 - x^2 - 1$  (quotient  $x^2$ )

Step 2:  $-x^3 + x^2 - 1$  (quotient  $-x$ )

Step 3:  $0 - x^2 + 1$  (quotient  $-1$ )

Step 4:  $0 - x^2 + x$  (quotient  $-1$ )

Step 5:  $0 - x + 1$  (quotient  $-1$ )

Step 6:  $0 - x + 1$  (quotient  $-1$ )

Final result:  $x^2 - x - 1$  with remainder  $-x + 1$ .

$\frac{x^3}{x} = x^2$

$\frac{-x^3}{x} = -x^2$

$\frac{-x}{x} = -1$

$\lim_{x \rightarrow 1} \frac{x^3 - 2x + 1}{(x-1)(x+2)}$

$= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 - x - 1)}{(x-1)(x+2)}$

$= \lim_{x \rightarrow 1} \frac{x^2 - x - 1}{x + 2} = \frac{1-1-1}{1+2} = \frac{-1}{3}$

Factorization of  $x^3 - 1 = (x-1)(x^2 + x + 1)$

Long division of  $x^3 - 2x^2 + 1$  by  $x - 1$  showing quotient  $x^2 - x - 1$  and remainder  $-x + 1$ .

- ① Direct sub.
- ② factorize
- ③ long division

Long division method

Synthetic Method:

③ Find:

a)  $\lim_{x \rightarrow 2} \frac{x^3 + 8}{x + 2}$

b)  $\lim_{x \rightarrow 4} \frac{2x - 8}{x^2 - x - 12}$

c)  $\lim_{x \rightarrow 2} \frac{x^3 - x^2 - 5x + 6}{x - 2}$

d)  $\lim_{x \rightarrow -3} \frac{x^3 - 10x - 3}{x^2 + 2x - 3}$

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Using Conjugate:

4 Find the following limits:

a  $\lim_{x \rightarrow 4} \frac{\sqrt{x-3} - 1}{x-4}$

b  $\lim_{x \rightarrow 5} \frac{x^2 - 5x}{\sqrt{x+4} - 3}$

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