

Relation between the two roots

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Sum and product of two roots

The two roots of the quadratic equation $ax^2 + bx + c = 0$ are:

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} a &= 1 \\ b &= 1 \\ c &= -2 \end{aligned}$$

let the first root be L and the other root be M then :

$$L + M = \frac{-b}{a} \quad (\text{Prove that})$$

$$LM = \frac{c}{a} \quad (\text{Prove that})$$

$$\begin{aligned} x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \\ \boxed{x = -2} \quad \boxed{x = 1} \end{aligned}$$

$$\begin{aligned} x_1 + x_2 &= \boxed{-1} \quad \text{Sum} \\ x_1 * x_2 &= \boxed{-2} \quad \text{Product} \end{aligned}$$

$$\boxed{L} + \boxed{M} = \frac{-b}{a} = \frac{-1}{1} = \boxed{-1}$$

$$LM = \frac{c}{a} = \frac{-2}{1} = \boxed{-2}$$

1 Without solving the equation find the sum and the product of the two roots of the equation: $2x^2 + 5x - 12 = 0$

$$ax^2 + bx + c = 0$$

$$L + M = \frac{-b}{a} = \frac{-5}{2}$$

$$L \cdot M = \frac{c}{a} = \frac{-12}{2} = \boxed{-6}$$

$$\begin{aligned} a &= 2 \\ b &= 5 \\ c &= -12 \end{aligned}$$

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2. If the product of the two roots of the equation $2x^2 - 3x + k = 0$ equals 1, find the value of k , then solve the equation.

$$\text{LM} = \frac{c}{a} = 1$$

$$2x^2 - 3x + k = 0$$

$$a = 2$$

$$b = -3$$

$$c = k$$

$$\frac{k}{2} = 1$$

$$\boxed{k = 2}$$

Relation between the two roots (Cont.1)

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3) If $(1+i)$ is one of the roots of the equation $x^2 - 2x + a = 0$ where $a \in \mathbb{R}$, then find:

A) The other root

B) the value of a .

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$L = 1+i$
 $a = 1$
 $b = -2$
 $C = a$

$L \cdot M = \frac{C}{a}$
 $L + M = \frac{b}{a}$

$1+i + M = \frac{2}{1}$
 $M = 2 - 1 - i$
 $M = 1 - i \rightarrow \text{other root.}$

$(1+i)(1-i) = \frac{a}{1}$
 $1 - i^2 = a$
 $a = 2 \neq 1$

Forming the quadratic equation whose two roots are known

$x^2 - (\text{sum})x + (\text{Product}) = 0$
 $x^2 - (L+M)x + LM = 0$
 $(x-4)(x+3) = 0$

$ax^2 + bx + c = 0$

4) Form the quadratic equation whose two roots are 4 and -3.

$x^2 - (L+M)x + LM = 0$

$L+M = 4 + (-3) = 1$

$L \cdot M = 4 \cdot (-3) = -12$

$x^2 - x + 12 = 0$

Relation between the two roots (Cont.2)

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$$x^2 - (L+M)x + LM = 0$$

5 Form the quadratic equation whose two roots are:

$$\frac{-2+2i}{1+i} \text{ and } \frac{-2-4i}{2-i}$$

$$L = \frac{-2+2i}{1+i} \quad \& \quad M = \frac{-2-4i}{2-i}$$

$$L+M = \frac{(-2+2i)(2-i) + (-2-4i)(1+i)}{(1+i)(2-i)}$$

$$= \frac{-4 - 2i^2 + 2i - 2i^2 + -2 - 2i - 4i - 4i^2}{2 - i + 2i - i^2}$$

$$= \frac{-4 - 2(-1) + 2i - 2(-1) + -2 - 2i - 4i - 4(-1)}{2 - i + 2i - (-1)}$$

$$= \frac{-4 + 2 + 2i - 2 + 2 + -2 - 2i - 4i + 4}{2 - i + 2i + 1}$$

$$= \frac{-2 - 4i}{3+i}$$

$$L+M = \frac{-6i}{3+i}$$

$$L = \frac{-2+2i}{1+i} \times \frac{1-i}{1-i} = \frac{-2+2i+2i-2i^2}{1-i^2}$$

CONJ

$$= \frac{4i + 2i^2}{2} = \frac{4i - 2}{2} = 2i - 1$$

$$M = \frac{-2-4i}{2-i} \times \frac{2+i}{2+i} = \frac{-4-2i-8i-4i^2}{4-i^2}$$

$$= \frac{-4-2i-8i+4}{4+1} = \frac{-10i}{5} = -2i = M$$

$$L+M = 2i + (-2i) = 0 \quad L \cdot M = 2i \cdot -2i = -4i^2 = 4$$

$$x^2 - 0x + 4 = 0 \quad x^2 + 4 = 0$$

Relation between the two roots (Cont.2)

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6 If L and M are the two roots of the equation $2x^2 - 3x - 1 = 0$, then form the quadratic equation whose roots are L^2 and M^2 .

$$L + M = \frac{-b}{a} = \frac{3}{2}$$

$$a = 2 \quad b = -3 \\ c = -1$$

$$L \cdot M = \frac{c}{a} = \frac{-1}{2}$$

$$\begin{aligned} L^2 + M^2 &= (L + M)^2 - 2LM \\ &= \left(\frac{3}{2}\right)^2 - 2 \cdot \frac{-1}{2} \\ &= \frac{9}{4} + 1 = \frac{13}{4} \end{aligned}$$

$$L^2 \cdot M^2 = (LM)^2 = \left(\frac{-1}{2}\right)^2$$

$$L^2 M^2 = \frac{1}{4}$$

$$x^2 - (L^2 + M^2)x + L^2 M^2 = 0$$

$$x^2 - \frac{13}{4}x + \frac{1}{4} = 0 \quad * 4$$

$$4x^2 - 13x + 1 = 0$$