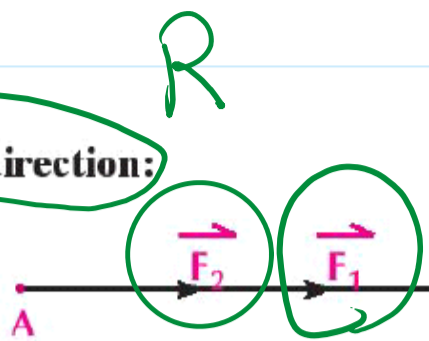


Special cases:

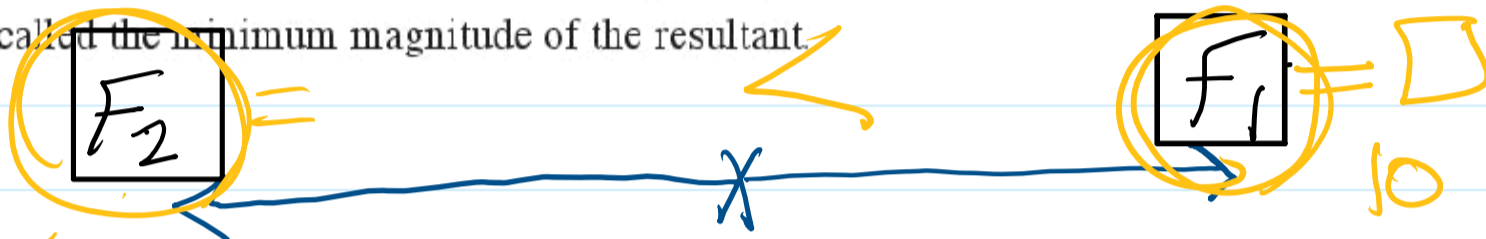
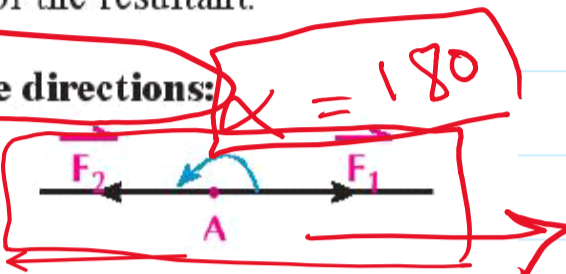
1- If the two forces have the same line of action and the same direction:

➤ In this case: $m(\angle \alpha) = 0$; $\cos \alpha = 1$ and by substitution in the resultant rule, we will find that $R = F_1 + F_2$, the direction of the resultant is the same direction as the two forces, In this case R is called the maximum magnitude of the resultant.



2- If the two forces have the same line of action and opposite directions:

➤ In this case: $m(\angle \alpha) = 180^\circ$; $\cos \alpha = -1$ and by substitution in the resultant rule, we will find that: $R = |F_1 - F_2|$ and \vec{R} acts in direction of the force or large magnitude, R is called the minimum magnitude of the resultant.



3 Two forces of magnitudes F , 4 newton act on a particle and the measure of the angle between their directions is 120° , the magnitude of their resultant equals $4\sqrt{3}$ newton. Find the magnitude of \vec{F} and the measure of the angle that \vec{R} form with \vec{F} .



$\theta = ?$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$

$(4\sqrt{3})^2 = (\sqrt{4^2 + F^2 + 2 \times 4 \times F \cos 120})^2$

$48 = 16 + F^2 + 8F \times -\frac{1}{2} \Rightarrow 32 = F^2 - 4F$

$F^2 - 4F - 32 = 0$
 $(F - 8)(F + 4) = 0$

$F = 8N$ or $F = -4$ refused

$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$

$\tan \theta = \frac{8 \sin 120}{4 + 8 \cos \alpha}$

$\tan \theta = \frac{\sqrt{3}}{3} \Rightarrow \theta = 30^\circ$

Resolution of a force into two given directions

Figure (1): shows resultant vector \vec{R} which is required to be resolved into two components \vec{F}_1, \vec{F}_2 in the two directions \vec{OA}, \vec{OB} which make angles of measure θ_1, θ_2 respectively with \vec{R}

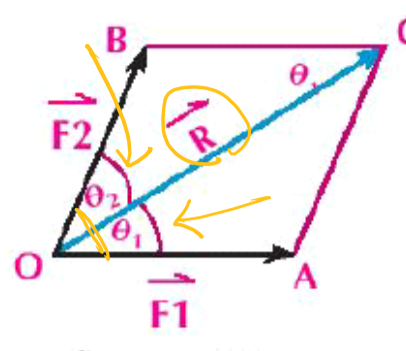


figure (1)

3rd

$$\sin(180 + \theta) = -\sin\theta$$

$$\sin(180 - \theta) = +\sin\theta$$

2nd

$$\sin(180 - \alpha) = \sin\alpha$$

$(\theta_1 + \theta_2)$ $(\theta_1 + \theta_2)$

Figure (2): shows triangle of forces where $\vec{AC} = \vec{OB}$

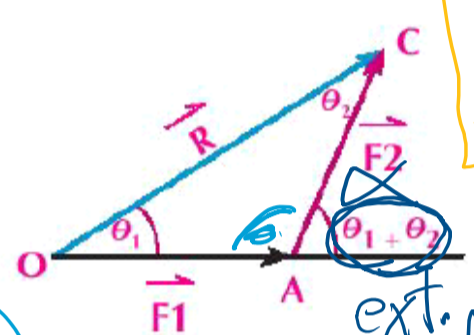


figure (2)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$\alpha = \theta_1 + \theta_2$

$180 - \alpha$

$$\sin(180 - \alpha) = \sin\alpha$$

$$= \sin(\theta_1 + \theta_2)$$

(From the properties of parallelogram) and by applying Sine rule:

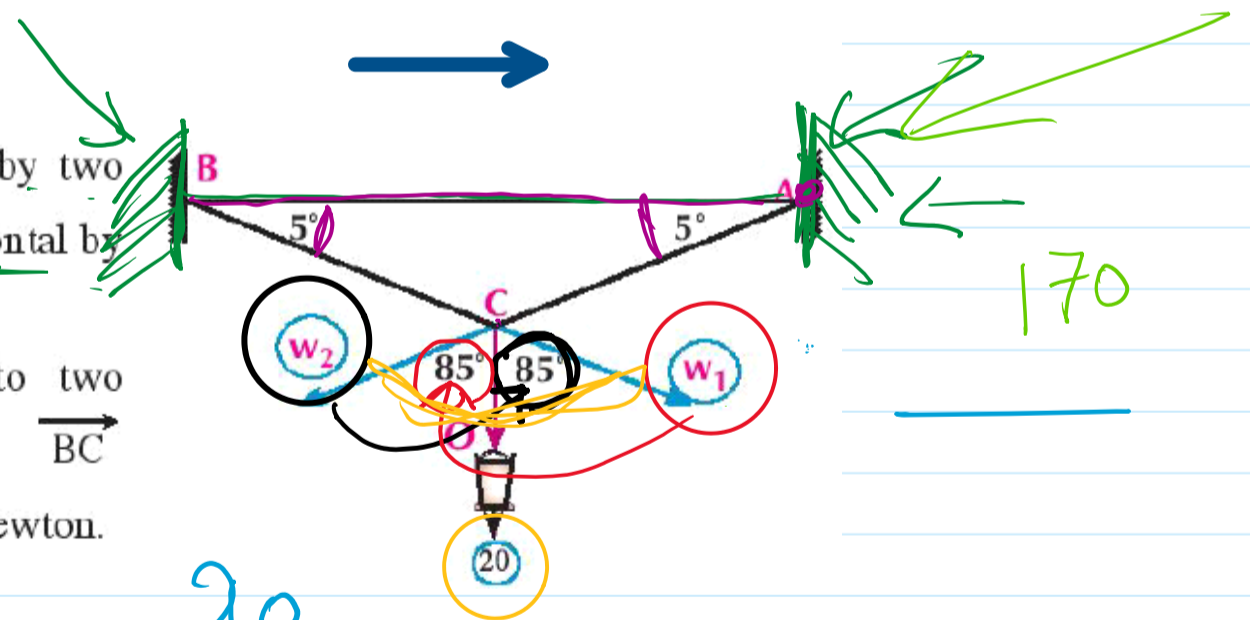
$$\frac{F_1}{\sin \theta_2} = \frac{F_2}{\sin \theta_1} = \frac{R}{\sin(\theta_1 + \theta_2)}$$

Notice that : $\sin(180 - (\theta_1 + \theta_2)) = \sin(\theta_1 + \theta_2)$

Example life applications

A lamp of weight 20 newtons suspended by two metal rods \vec{AC}, \vec{BC} inclined to the horizontal by two equal angles, the measure of each is 5° .

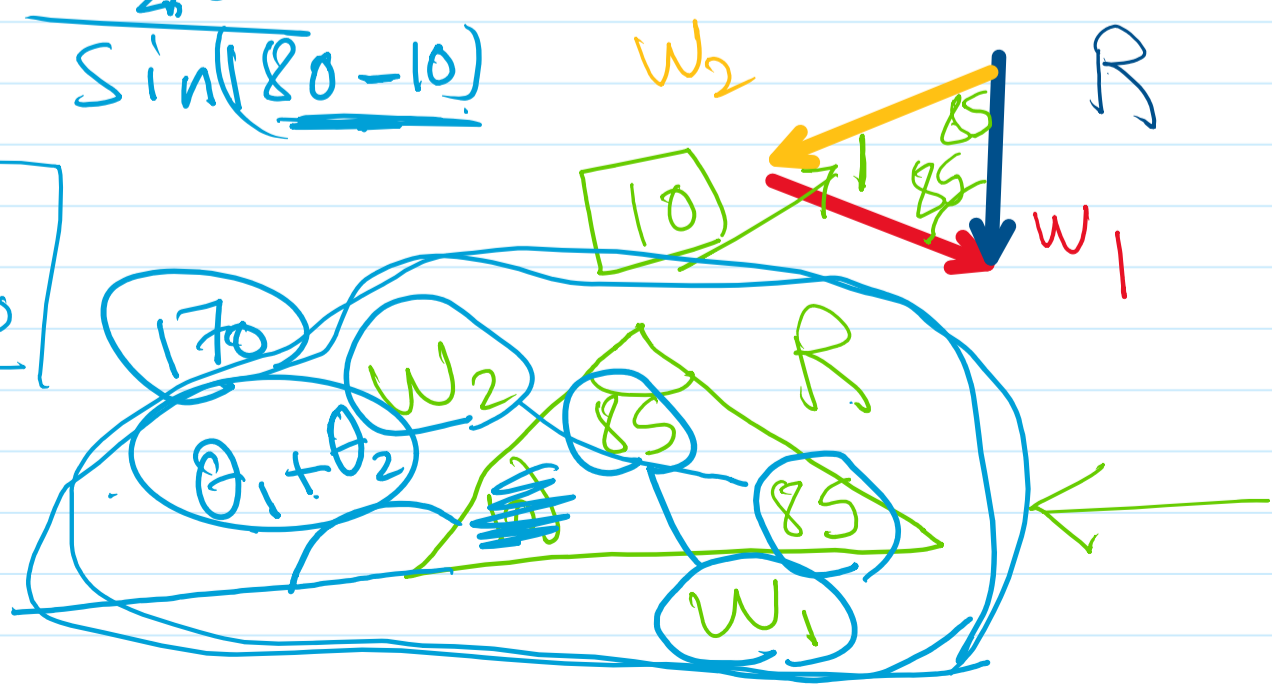
Resolve the weight of the lamp into two components in the direction \vec{AC}, \vec{BC} approximating the result to the nearest newton.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{20}{\sin(180 - 10)}$$

$$\frac{W_2}{\sin 85} = \frac{W_1}{\sin 85} = \frac{20}{\sin 170}$$

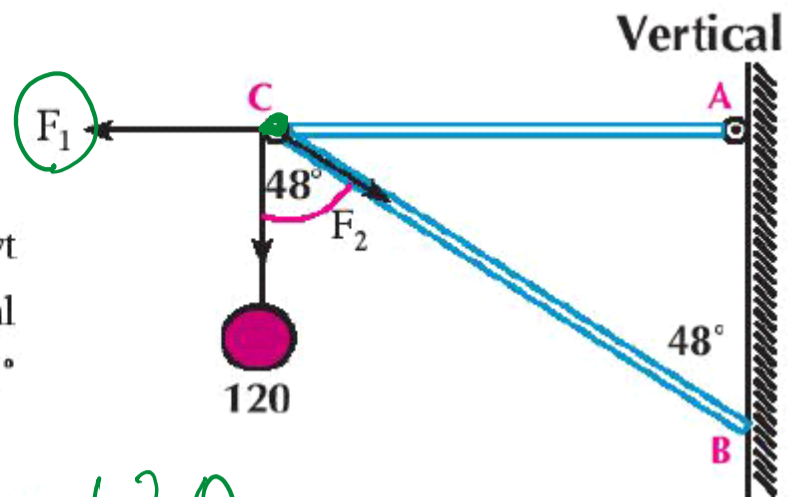


Justify your answer.

Try To Solve

2 In the opposite figure:

Resolve the vertical force of magnitude 120gm. wt into two components one of them is the horizontal and the other inclined by an angle of measure 48° to the line of action of the force.



$$\frac{F_1}{\sin 48} = \frac{F_2}{\sin 90} = \frac{120}{\sin(48+90)}$$

$$F_1 = \frac{F_2 \sin 48}{\sin \theta}$$

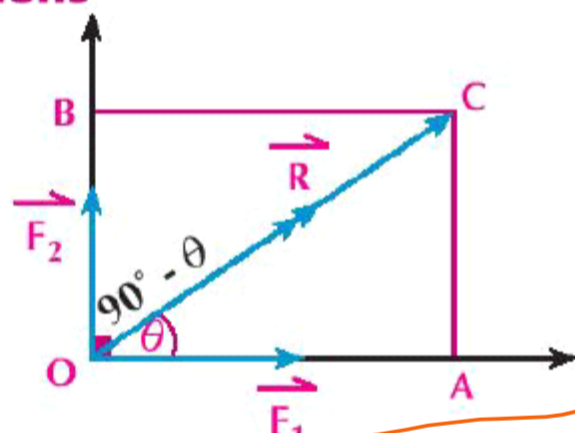
$$F_1 = \frac{120 \sin 48}{\sin(48+90)}$$

$F_1 = \checkmark$

$$F_2 = \frac{F_1 \sin \theta}{\sin 48}$$

Resolution of a force into two perpendicular directions

If \vec{R} acts on a particle (O) as in the opposite figure and their perpendicular components are \vec{F}_1 , \vec{F}_2 where \vec{F}_1 is inclined by an angle of measure θ with \vec{R} , then the parallelogram will be a rectangle ACBO, by applying sine rule on triangle OAC then:



$$\frac{F_1}{\sin(90^\circ - \theta)} = \frac{F_2}{\sin \theta} = \frac{R}{\sin 90^\circ} \therefore \frac{F_1}{\cos \theta} = \frac{F_2}{\sin \theta} = R$$

$$F_1 = \frac{F_2 \cos \theta}{\sin \theta}$$

$$\sin(90 - \theta) = \cos \theta$$

270

Inclined Plane

It is a surface that inclines to the horizontal plane by an angle whose measure is θ , $0 < \theta < \frac{\pi}{2}$ as shown in the opposite figure

