

The Cosine Rule

$\triangle BDC$ is a right angled at D:

$(BC)^2 = (CD)^2 + (BD)^2$ (pythagoras theorem)

then:

$a^2 = (b \sin A)^2 + (c - b \cos A)^2$
 $= b^2 \sin^2 A + b^2 \cos^2 A + c^2 - 2bc \cos A$

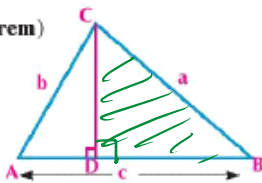
(Expanding brackets)

$= b^2 (\sin^2 A + \cos^2 A) + c^2 - 2bc \cos A$ (b^2 common factor)

$= b^2 + c^2 - 2bc \cos A$ (simplifying)

then:

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



Remember
 Pythagoras identity
 $\sin^2 A + \cos^2 A = 1$

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

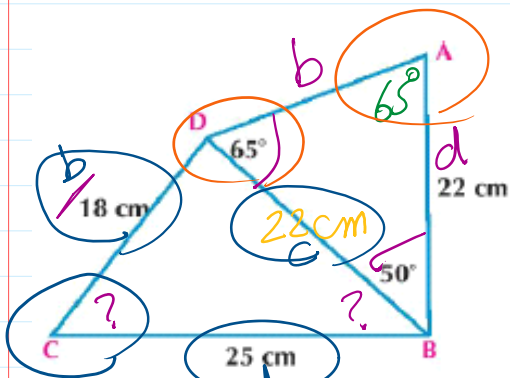
$c^2 = a^2 + b^2 - 2ab \cos C$

$\frac{2abc \cos C}{2ab} = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

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- 5 ABCD is a quadrilateral in which $AB = 22\text{cm}$, $m(\angle ADB) = 65^\circ$, $m(\angle DBA) = 50^\circ$, $BC = 25\text{cm}$, $DC = 18\text{cm}$, Find: $m(\angle CBD)$, $m(\angle BCD)$



$180 - (65^\circ + 50^\circ) = 65^\circ$

$\frac{a}{\sin 65} = \frac{b}{\sin 50} = \frac{22}{\sin 65}$

$a = \frac{22 \times \sin 65}{\sin 50}$ $a = 22\text{cm}$

$\cos C = \frac{d^2 + b^2 - a^2}{2db} = \square$

$m(\angle C) = \cos^{-1} \square = 79.4$

$\cos B = \frac{c^2 + d^2 - b^2}{2cd} = \square$

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- 6 ABCD is a quadrilateral in which $AB = 3\text{cm}$, $AC = 8\text{cm}$, $BC = 7\text{cm}$, $CD = 5\text{cm}$, $BD = 8\text{cm}$, prove that the shape ABCD is a cyclic quadrilateral.

$m(\angle A) + m(\angle C) = 180^\circ$

$m(\angle B) + m(\angle D) = 180^\circ$

$m(\angle CDB) = m(\angle BAC)$ opp. to same base \overline{CB}
 $b^2 + c^2 - d^2$

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$\checkmark m(\angle CDB) = m(\angle BAC)$ opp. To same base

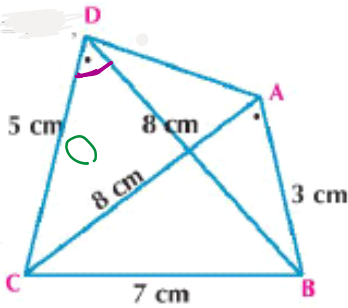
$$\cos D = \frac{b^2 + c^2 - d^2}{2bc} =$$

$\square CB$

$$m(\angle D) = 60^\circ$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} =$$

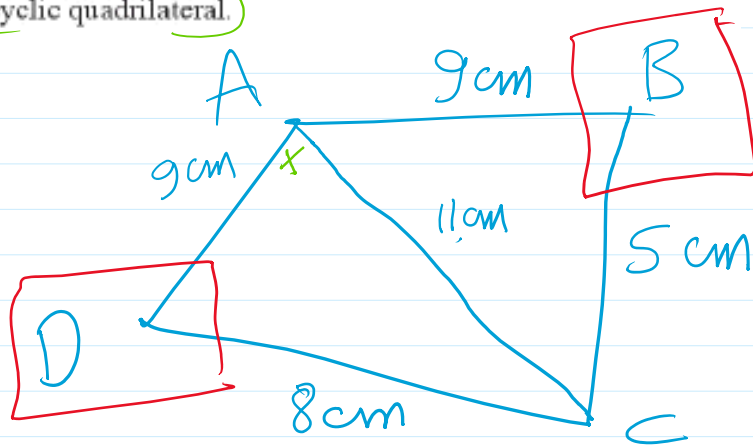
$$m(\angle A) = 60^\circ$$



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15 ABCD is a quadrilateral in which $AB = 9$ cm, $BC = 5$ cm, $CD = 8$ cm, $AD = 9$ cm, $AC = 11$ cm, prove that the shape ABCD is a cyclic quadrilateral.

$$\cos A = \frac{d^2 + c^2 - a^2}{2dc}$$



The Cosine Rule

$\triangle BDC$ is a right angled at D:

$(BC)^2 = (CD)^2 + (BD)^2$ (pythagoras theorem)

then:

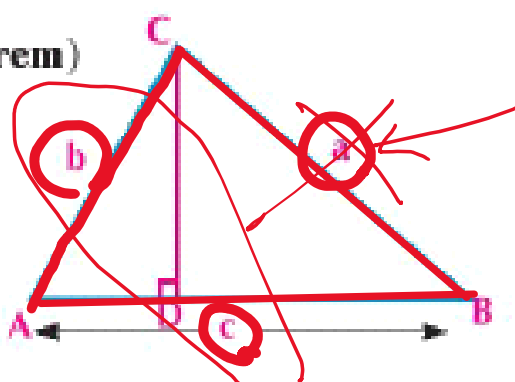
$$a^2 = (b \sin A)^2 + (c - b \cos A)^2$$

$$= b^2 \sin^2 A + b^2 \cos^2 A + c^2 - 2bc \cos A$$

(Expanding brackets)

$$= b^2 (\sin^2 A + \cos^2 A) + c^2 - 2bc \cos A \quad (b^2 \text{ common factor})$$

$$= b^2 + c^2 - 2bc \cos A \quad (\text{simplifying})$$



then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Remember

Pythagoras identity

$$\sin^2 A + \cos^2 A = 1$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = b^2 + a^2 - 2ab \cos C$$

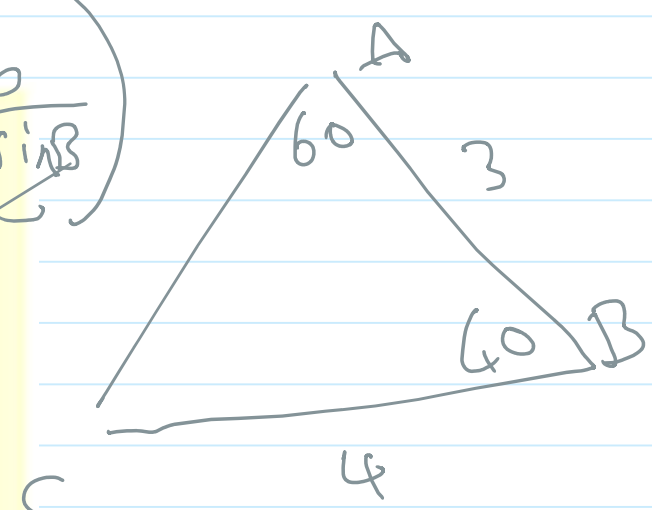
The cosine rule provides that :

In any triangle ABC:

$$a^2 = b^2 + c^2 - 2bc \cos A, \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

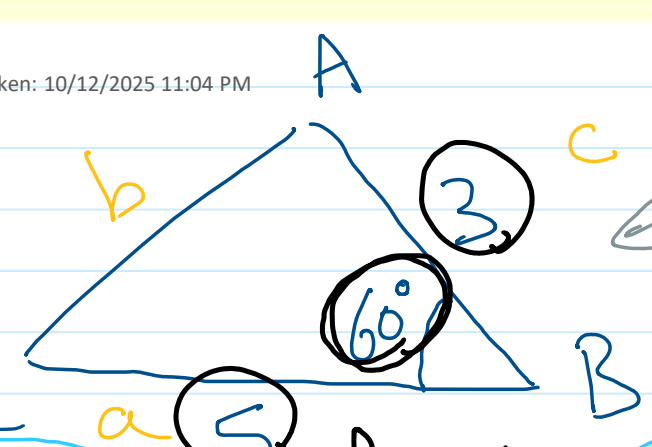
$$b^2 = c^2 + a^2 - 2ca \cos B, \quad \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$c^2 = a^2 + b^2 - 2ab \cos C, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



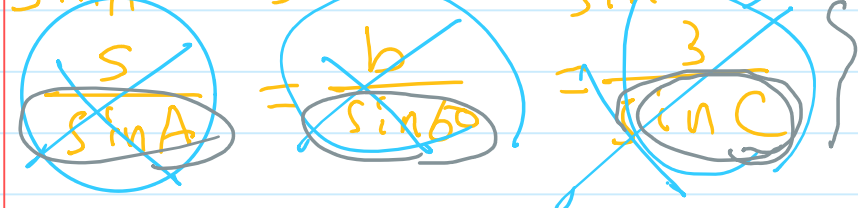
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

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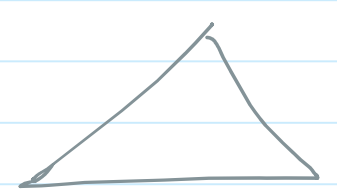
① 2 sides & angle bet. \rightarrow cos. rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

② if all sides are given only



Trigonometry (Cosine Rule)_2

Sunday, October 12, 2025 11:02 PM

① Find the measure of the **greatest angle** of the triangle ABC in which $a = 4.6$, $b = 3.2$, $c = 2.8$

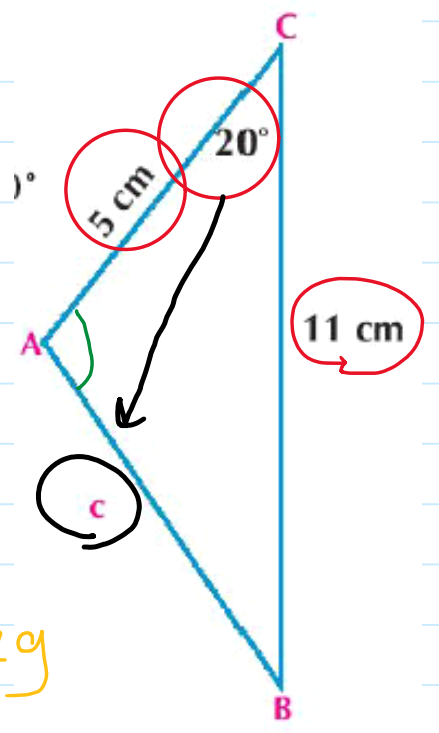
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(3.2)^2 + (2.8)^2 - (4.6)^2}{2 \times 3.2 \times 2.8} = \frac{-11}{64}$$

$$m(\angle A) = 99^\circ 53' 48.7''$$

② Solve the triangle ABC in which $a = 11 \text{ cm}$, $b = 5 \text{ cm}$, $m(\angle C) = 20^\circ$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

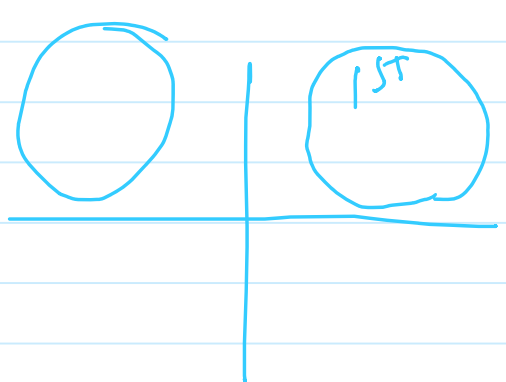
$$c = \sqrt{11^2 + 5^2 - 2 \times 11 \times 5 \cos 20} = 6.5 \text{ cm}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{11}{\sin A} = \frac{6.5}{\sin 20}$$

$$\sin A = \frac{11 \sin 20}{6.5} = 0.579$$



$$m(\angle A) = 35^\circ$$

$$m(\angle A) = 145^\circ$$

$180 - 35$
 $\therefore BC$ is greatest side
 $\therefore \angle A$ is the ~ angle
 $\therefore m(\angle A) = 180 - 35 = 145^\circ$

Trigonometry (Cosine Rule)_3

Sunday, October 12, 2025 11:08 PM

- 4) ABC is a triangle in which $a = 63 \text{ cm}$, $b - c = 27 \text{ cm}$, the perimeter of the triangle is 140 cm , Find each of b , c and the measure of the smallest angle of the triangle also find the area of the triangle to the nearest centimeter square.

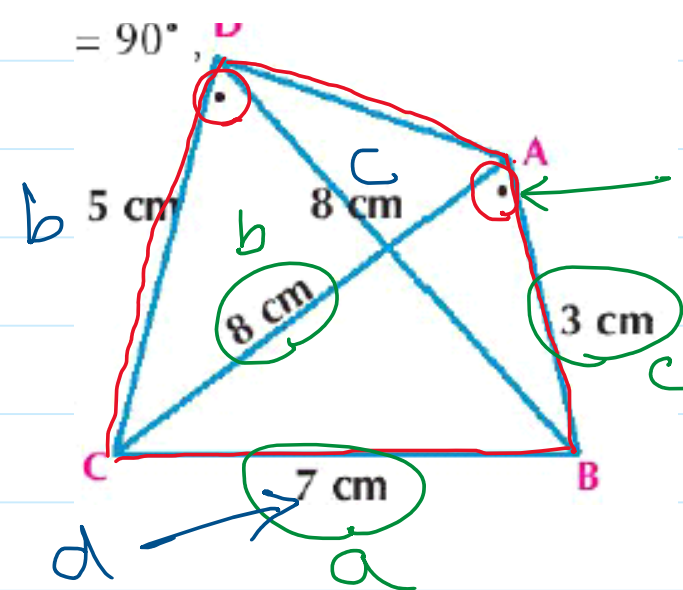
- 6) ABCD is a quadrilateral in which $AB = 3 \text{ cm}$, $AC = 8 \text{ cm}$, $BC = 7 \text{ cm}$, $CD = 5 \text{ cm}$, $BD = 8 \text{ cm}$, prove that the shape ABCD is a cyclic quadrilateral.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 3^2 - 7^2}{2 \times 8 \times 3} = \frac{1}{2}$$

$$m(\angle A) = 60^\circ$$

$$\text{C.} \quad \cos D = \frac{b^2 + c^2 - d^2}{2bc} = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8} = \frac{1}{2}$$

$$m(\angle D) = 60^\circ$$



$\therefore m(\angle BAC) = m(\angle CDB)$
 $\therefore \overline{CB}$ is common base
 $\therefore ABCD$ is cyclic quad.

Trigonometry (Cosine Rule)_4

Sunday, October 12, 2025 11:16 PM

- 5 ABCD is a quadrilateral in which $AB = 22\text{cm}$, $m(\angle ADB) = 65^\circ$, $m(\angle DBA) = 50^\circ$, $BC = 25\text{cm}$, $DC = 18\text{cm}$, Find: $m(\angle CBD)$, $m(\angle BCD)$

