

Limit of the functions at infinity_1

Monday, October 13, 2025 7:00 PM

Theorem 1

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

Corollary

$$\lim_{x \rightarrow \infty} \frac{a}{x^n} = 0 \quad \text{where } n \in \mathbf{R}^+, a \text{ is constant}$$

$$\boxed{\infty} + \infty = \boxed{\infty}$$

$$\infty + 0 = \boxed{\infty}$$

$$\frac{2}{\infty} = 0 \quad \checkmark \quad \frac{5}{\infty} = 0 \quad \checkmark$$

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1 Find:

a $\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 3\right)$

b $\lim_{x \rightarrow \infty} \left(4 - \frac{3}{x^2}\right)$

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$$\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 3\right) = \frac{1}{\infty} + 3 = 0 + 3 = \boxed{3}$$

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2 Find: $\lim_{x \rightarrow \infty} (4x - x^3 + 5)$

$\infty - \infty + 5 = \cancel{\infty}$

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$$\begin{aligned} \lim_{x \rightarrow \infty} (4x - x^3 + 5) &= \lim_{x \rightarrow \infty} x^3 \left(\frac{4}{x^2} - 1 + \frac{5}{x^3} \right) = \infty \left(\frac{4}{\infty} - 1 + \frac{5}{\infty} \right) \\ &= \infty * -1 = \boxed{-\infty} \end{aligned}$$

Limit of the functions at infinity_2

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3 Find each of the following limits:

a $\lim_{x \rightarrow \infty} \frac{2x-3}{3x^2+1}$

b $\lim_{x \rightarrow \infty} \frac{2x^2-3}{3x^2+1}$

c $\lim_{x \rightarrow \infty} \frac{2x^3-3}{3x^2+1}$

$\lim_{x \rightarrow \infty} \frac{2x^2-3}{3x^2+1} = \lim_{x \rightarrow \infty} \frac{x^2(2-\frac{3}{x^2})}{x^2(3+\frac{1}{x^2})} = \lim_{x \rightarrow \infty} \frac{2-\frac{3}{x^2}}{3+\frac{1}{x^2}}$

$= \frac{2-0}{3+0} = \frac{2}{3}$

a $\lim_{x \rightarrow \infty} \frac{2x-3}{3x^2+1} = \lim_{x \rightarrow \infty} \frac{x(2-\frac{3}{x})}{x^2(3+\frac{1}{x^2})} = \frac{1}{\infty} = 0$

Power in den. > P. in num. $\rightarrow \lim_{x \rightarrow \infty} \frac{x^2(2-\frac{3}{x^2})}{x^2(3+\frac{1}{x^2})} = \frac{0-0}{3-0} = \frac{0}{3} = 0$

~~$\frac{x^3(2-\frac{3}{x^3})}{x^3(3+\frac{1}{x^3})} = \frac{2-0}{0-0} = \text{ERROR}$~~

4 Find the following limits:

a $\lim_{x \rightarrow \infty} \frac{x^3-2}{|x|^3+1}$

b $\lim_{x \rightarrow \infty} (x - \sqrt{x^2+4})$

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$|x| = x$

$\lim_{x \rightarrow \infty} \frac{x^3-2}{x^3+1} = \lim_{x \rightarrow \infty} \frac{x^3(1-\frac{2}{x^3})}{x^3(1+\frac{1}{x^3})} = 1$

Limit of the functions at infinity_3

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4 Find the following limits :

a $\lim_{x \rightarrow \infty} \frac{x^3 - 2}{|x|^3 + 1}$

b $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + 4})$

$$\lim_{x \rightarrow \infty} \boxed{x} - \sqrt{x^2 + 4} \rightarrow \frac{x + \sqrt{x^2 + 4}}{x + \sqrt{x^2 + 4}} =$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 + 4)}{x + \sqrt{x^2 + 4}} = \lim_{x \rightarrow \infty} \frac{\cancel{x^2} - \cancel{x^2} - 4}{x + \sqrt{x^2 + 4}}$$

$$= \lim_{x \rightarrow \infty} \frac{x \cdot (-4/x)}{x + \sqrt{x^2 + 4}} = \lim_{x \rightarrow \infty} \frac{x \cdot (-4/x)}{x + \sqrt{x^2 (1 + 4/x^2)}}$$

$$= \lim_{x \rightarrow \infty} \frac{x \cdot (-4/x)}{x + \sqrt{x^2} \sqrt{1 + 4/x^2}} = \lim_{x \rightarrow \infty} \frac{x \cdot (-4/x)}{x + x \sqrt{1 + 4/x^2}}$$

$$\lim_{x \rightarrow \infty} \frac{\cancel{x} \cdot (-4/x)}{\cancel{x} (1 + \sqrt{1 + 4/x^2})} = \frac{0}{1 + \sqrt{1 + 0}} = \frac{0}{1 + 1} = \frac{0}{2} = \boxed{0}$$