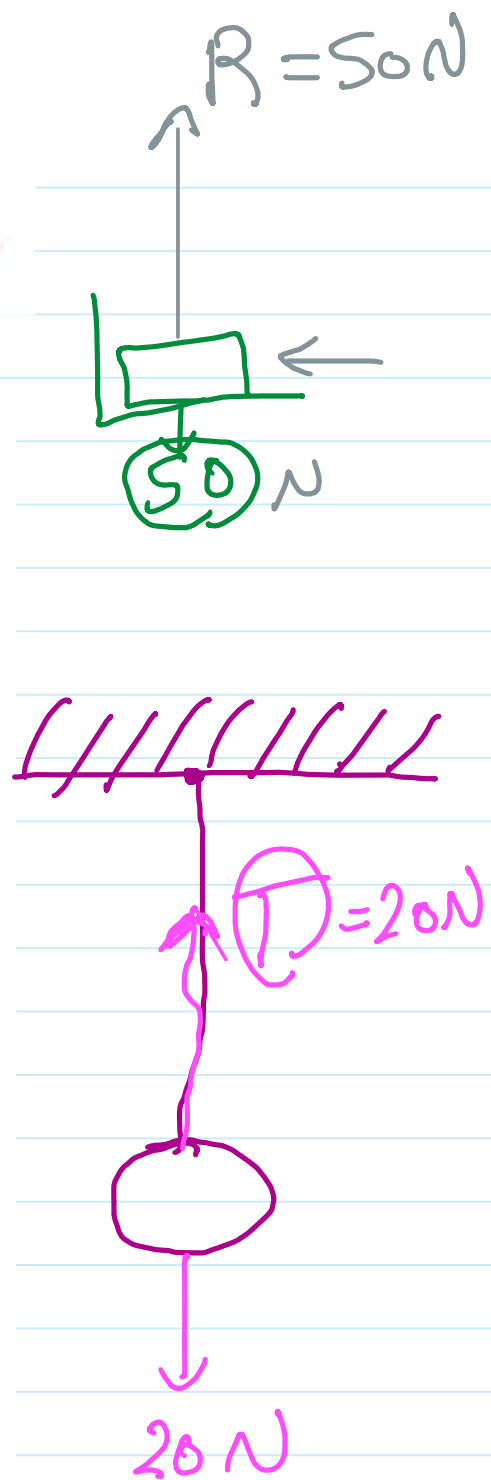
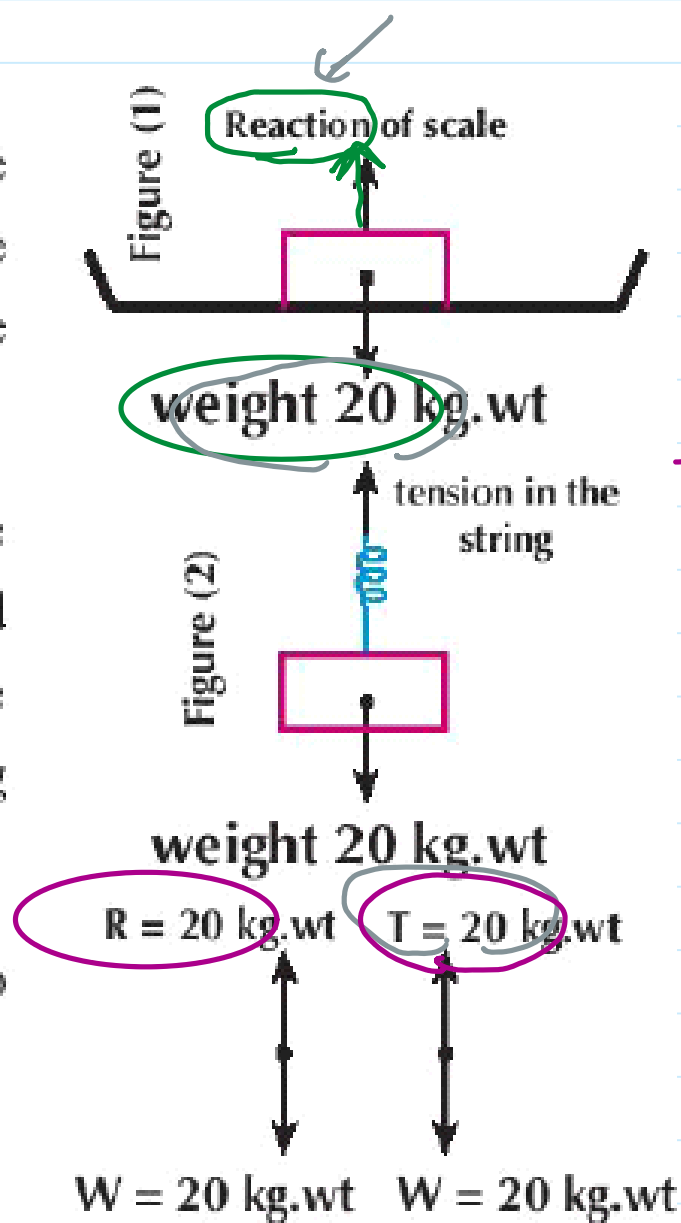


Equilibrium of a rigid body under the effect of two forces

- 1- Put a body of weight 20 kg.wt on the scale pan of a horizontal pressure balance and notice the reading of the balance. figure (1)
- 2- Ask your classmate to suspend the same body by a light smooth string and the end of the string by a hook of spring balance then to notice the reading of the spring balance in the of case of rest. figure (2)
- 3- Compare the results in the two experiments. What do you notice?

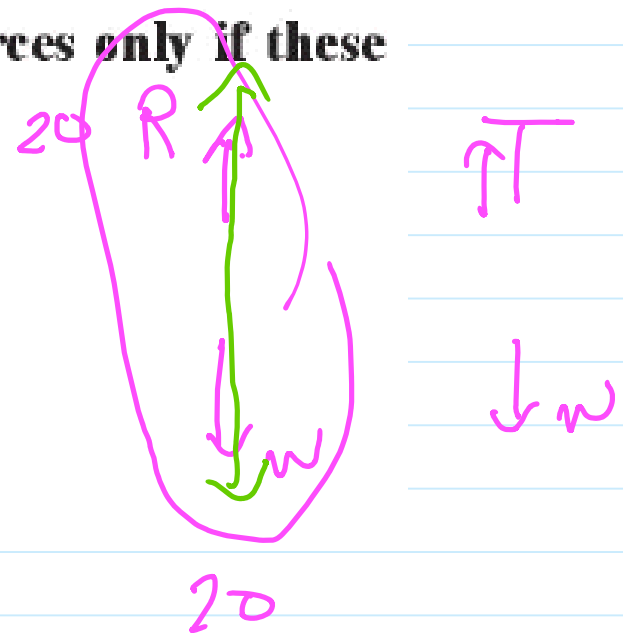
Notice:



Terms of balancing a rigid body under the effect of two forces

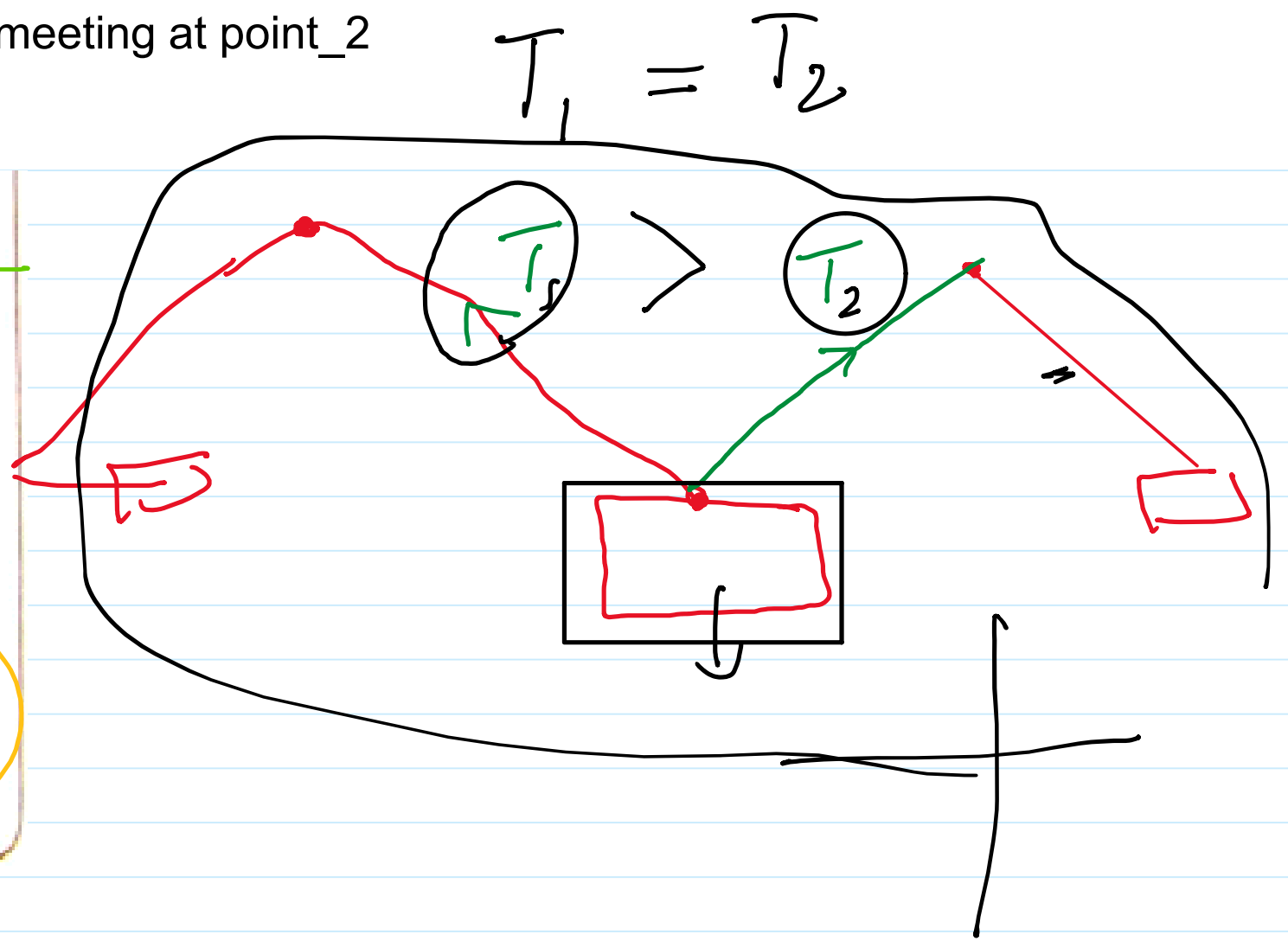
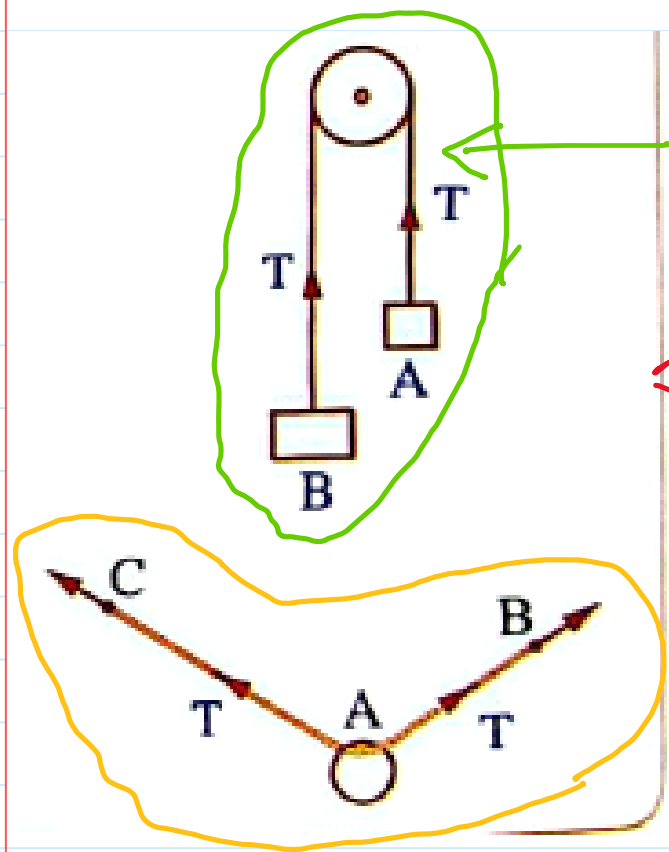
A rigid body is balanced under the effect of two forces only if these two forces are:

- 1- equal in magnitude.
- 2- opposite in direction.
- 3- their lines of action are on the same straight line.



Equilibrium of a rigid body meeting at point_2

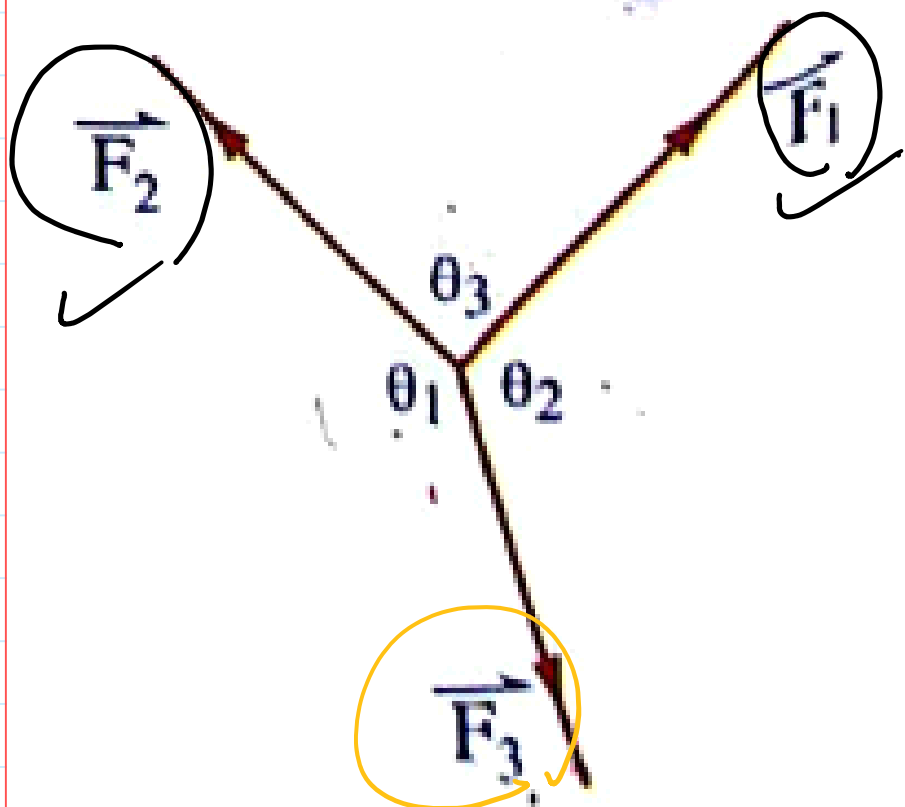
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Find the resultant forces for each two forces:

\vec{F}_1 , \vec{F}_2 and \vec{F}_3 are three coplanar forces meeting at a point, their magnitudes are 12, $12\sqrt{3}$ and 24 newton respectively. If these forces are balanced, find the measures of the angles among the three lines of action of the three forces.

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$$R_{1,2} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta_3}$$

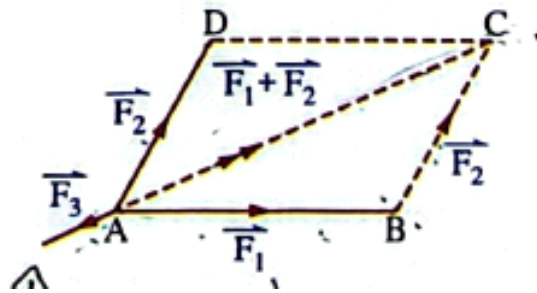
$$24^2 = 12^2 + (12\sqrt{3})^2 + 2 \times 12 \times (12\sqrt{3}) \cos \theta_3$$

Equilibrium of a rigid body meeting at point_3

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Rule

If three forces are acting at a point and can be represented by the sides of a triangle taken in the same cyclic order, then the forces are in equilibrium.



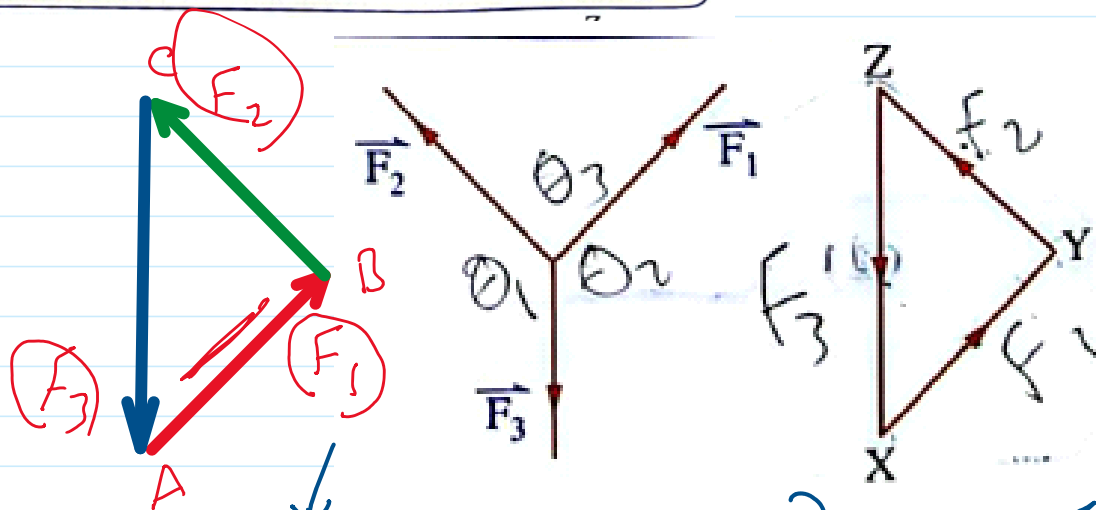
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Rule (2) The triangle of forces rule

If a rigid body is in equilibrium under the action of three forces acting at a point and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order, then the lengths of the sides of the triangle are proportional to the magnitudes of the corresponding forces.

$$\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{CA}$$



Three forces of magnitudes F_1 , F_2 and 96 kg.wt. act on a particle. They are represented by the directed line segments \overline{AB} , \overline{BC} and \overline{CA} respectively in ΔABC , where $AB = 8 \text{ cm.}$, $BC = 10 \text{ cm.}$ and $CA = 12 \text{ cm.}$ Find the value of each of F_1 and F_2 .

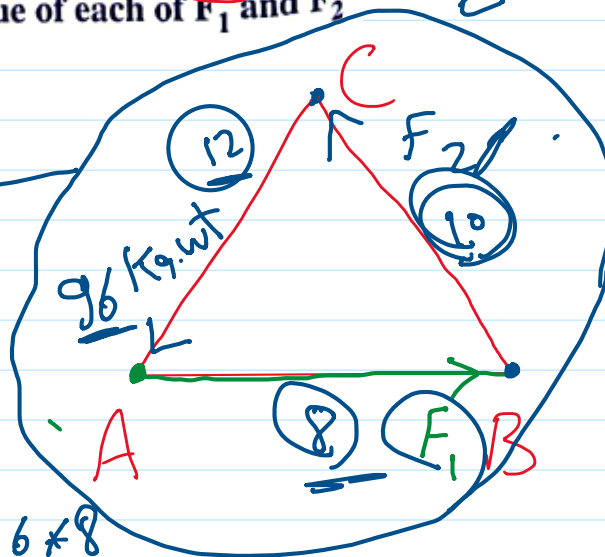
$$\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{CA}$$

$$\frac{F_1}{8} = \frac{F_2}{10} = \frac{96}{12}$$

$$F_2 = \frac{10 \times 96}{12} = 80$$

$$F_2 = 80$$

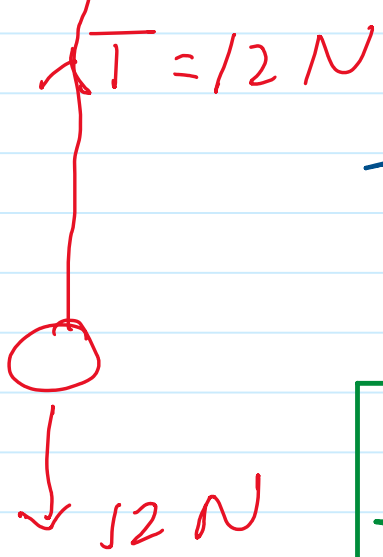
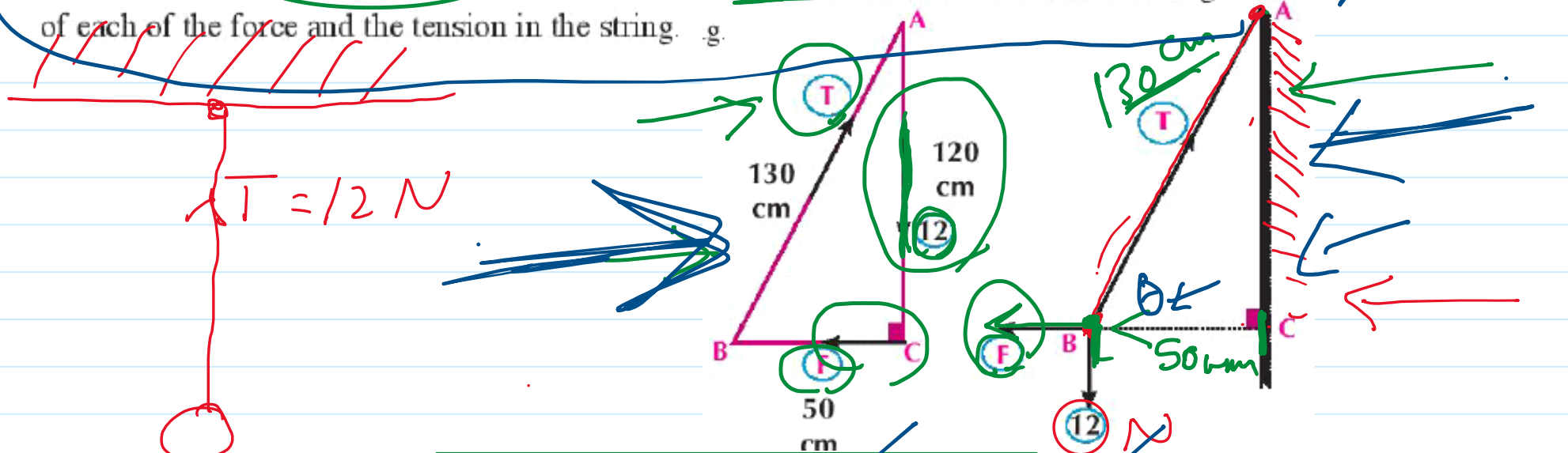
$$F_1 = \frac{96 \times 8}{12} = 64$$



Equilibrium of a rigid body meeting at point_4

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3 A body of weight 12 Newton is hanged from one end of a light string whose length 130 cm. The other end is fixed at a point in the vertical wall the body is pulled by a horizontal force which makes the body in equilibrium when it is at distance 50 cm from the wall. Find the magnitude of each of the force and the tension in the string. g.



$$\frac{T}{130} = \frac{12}{120} = \frac{F}{50}$$

$$130^2 = 50^2 + AC^2$$

$$AC = \sqrt{130^2 - 50^2}$$

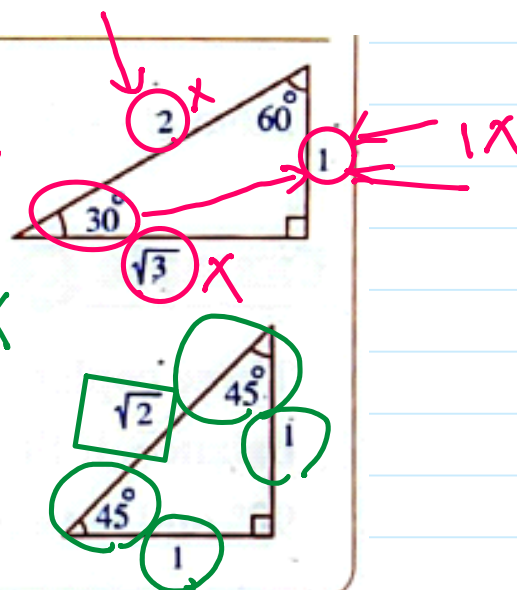
$$= 120$$

3 T F

Enrich your Knowledge

* If the triangle of forces of three equilibrium forces is a right-angled triangle and it has an angle of measure 30° , then the ratio among its side lengths is $1 : 2 : \sqrt{3}$

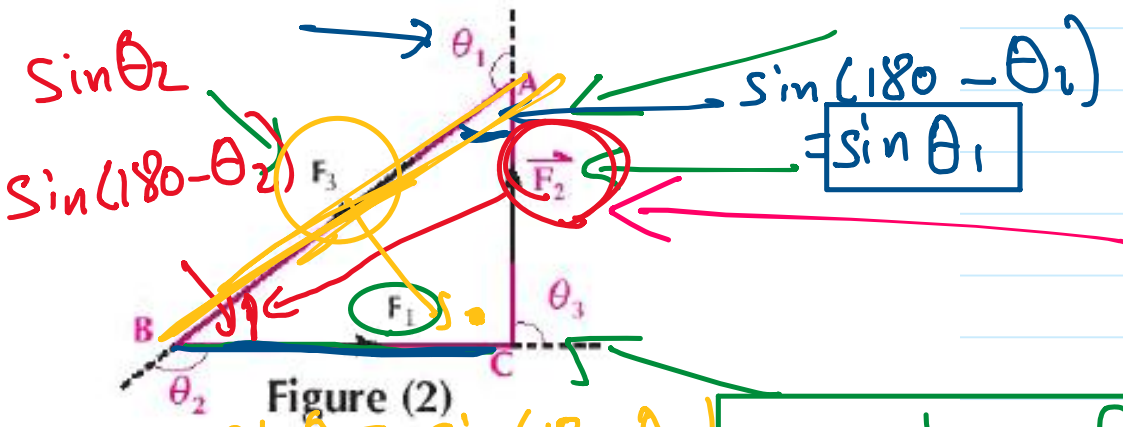
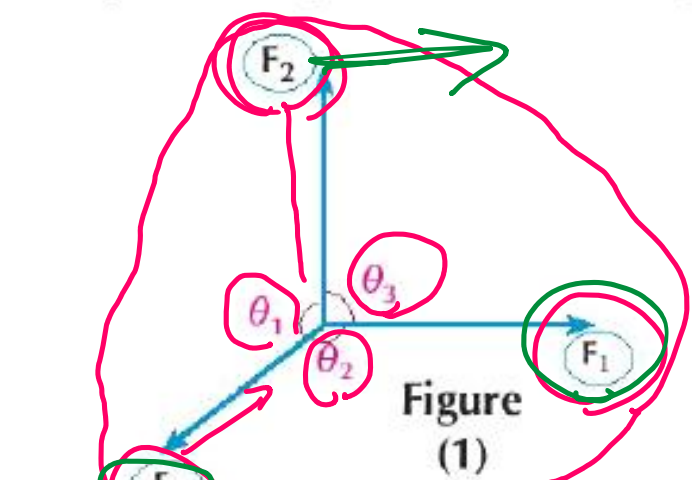
$1x : 2x : \sqrt{3}x$



* If the triangle of forces is isosceles right-angled triangle, then the ratio among its side lengths is $1 : 1 : \sqrt{2}$

Lami's rule:

If the coplanar forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ act at a point and equilibrium as in figure (1), then it could be represented by the sides of a triangle taken in the same cyclic order as in figure (2)



Using the sine law:

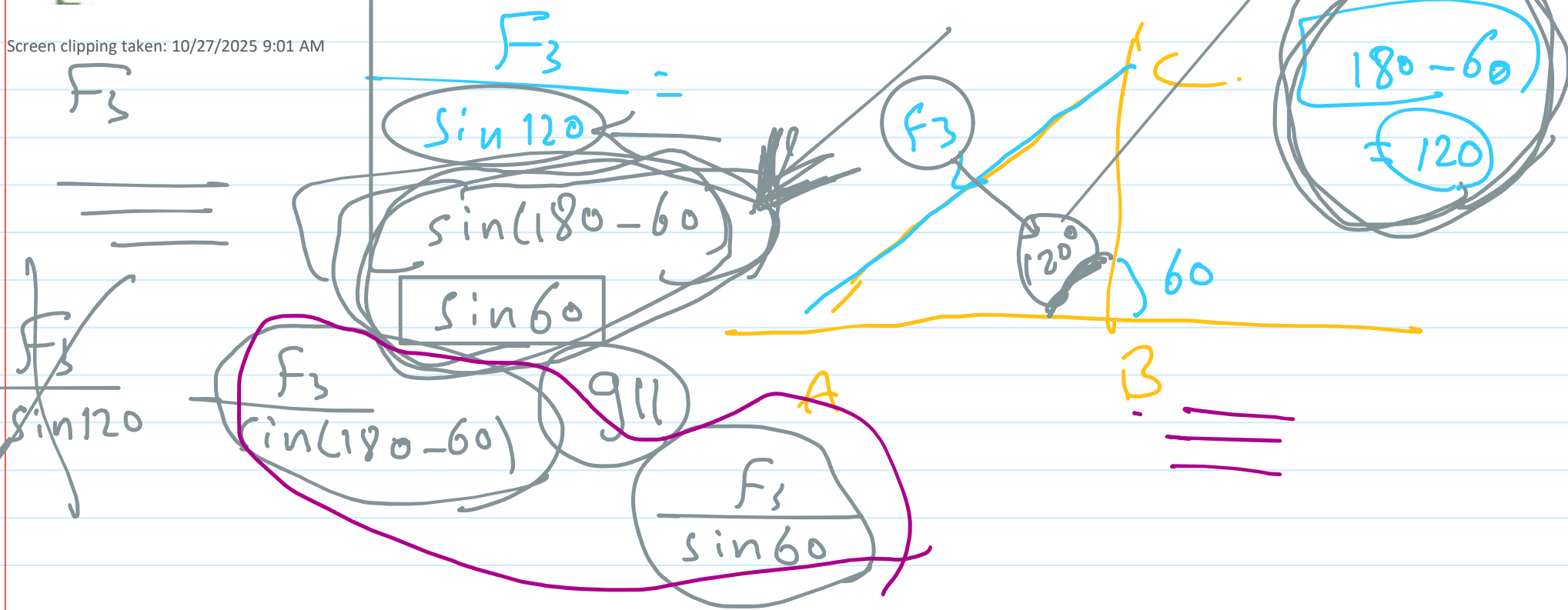
$$\frac{BC}{\sin(180 - \theta_1)} = \frac{CA}{\sin(180 - \theta_2)} = \frac{AB}{\sin(180 - \theta_3)}$$

i.e. $\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

If a body is in equilibrium under the effect of three forces meeting at a point, then the magnitude of each force is proportional to the sine of the angle between the two other forces

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Equilibrium of a rigid body meeting at point_6

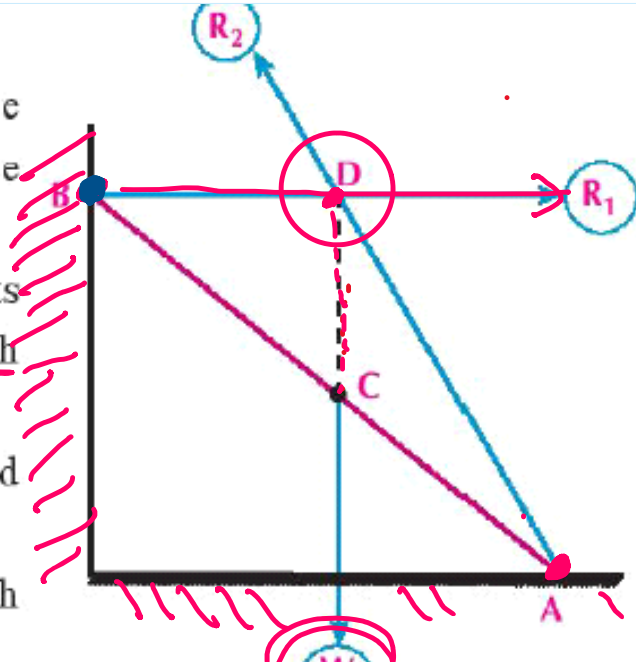
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Rule:

If a rigid body is in equilibrium under the effect of three coplanar non parallel forces, then the lines of action of the three forces meet at one point.

For example: If a uniform rod of weight w rests with its end B on a smooth vertical wall and with its end A on rough horizontal ground, then:

- The weight of the rod acts at the midpoint of the rod vertically downwards.
- Reaction of the wall R_1 is perpendicular to the smooth wall in the direction BD .
- Reaction of the rough ground R_2 its direction is not defined, so to find its direction, draw \vec{AD} which passes through the point D (point of intersection of the two lines of action of \vec{W} and \vec{R}_1) as in the figure.



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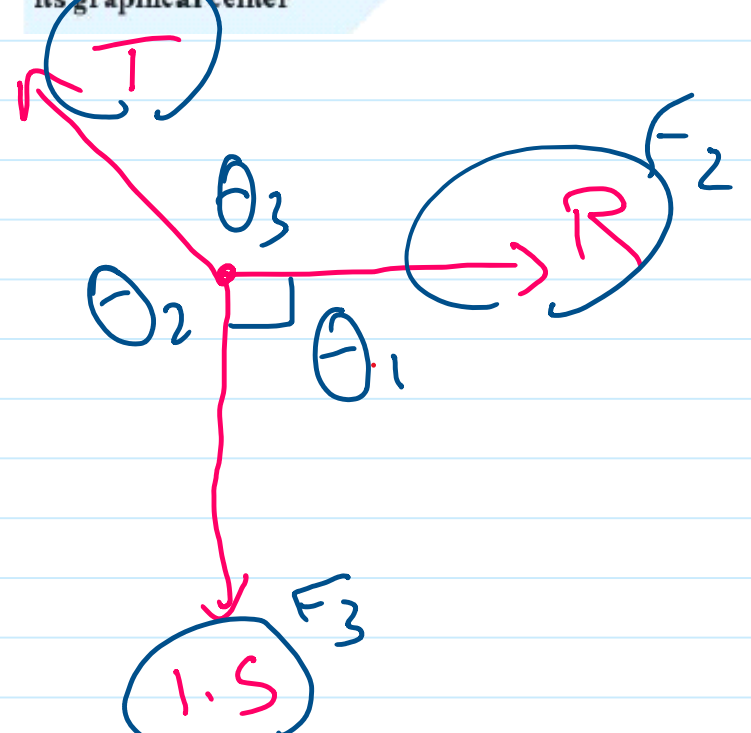
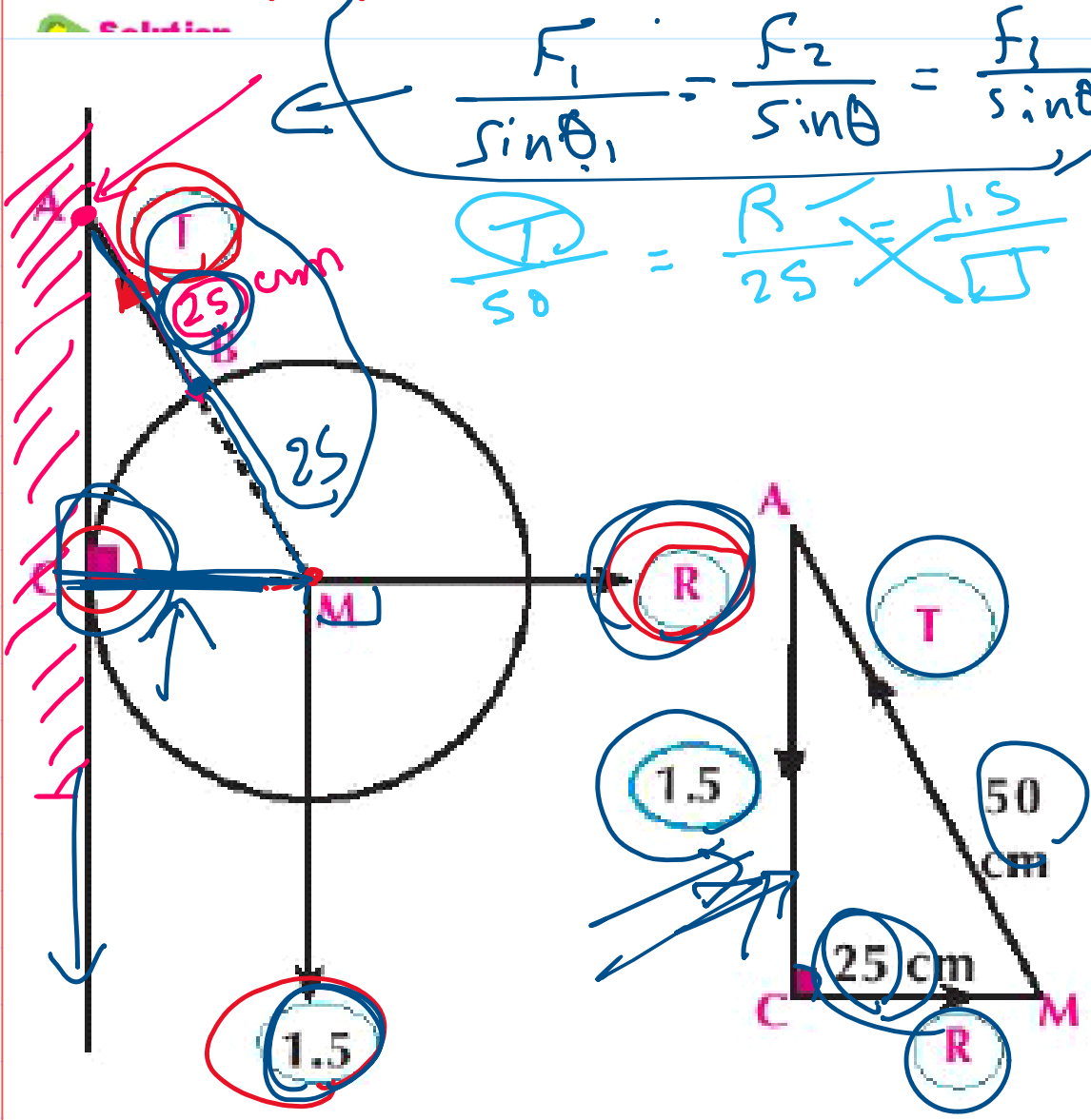
- 5 A metallic smooth regular sphere of weight 1.5 kg .wt. and radius length 25 cm is suspended from a point B on its surface by a string of length 25 cm. Its other end A is fixed at a point in a smooth vertical wall to be in equilibrium as it rests on the wall. Find the magnitude of the tension in the string and the reaction of the wall.

Remember that

The center of gravity of a homogenous sphere is its graphical center

$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta} = \frac{F_3}{\sin \theta_2}$$

$$\frac{T}{50} = \frac{R}{25} = \frac{1.5}{\sqrt{3}}$$



$$AC = \sqrt{50^2 - 25^2} = \dots$$

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Equilibrium of a rigid body meeting at point_7

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Example

- 6 A uniform rod of length 100 cm and weight 30 N is suspended at its ends freely by two perpendicular strings, their two ends are fixed at a hook. If the length of one string is 50 cm. Find the magnitude of the tension in each of the two strings. When the rod is hanged freely and in equilibrium.

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