

Solving absolute Value Equations and Inequalities_1

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1 Solve the equation: $|x - 3| = 5$ graphically and algebraically.

$f(x)$ ← $|x - 3|$ → $g(x)$

$f(x) = |x - 3|$ → abs.

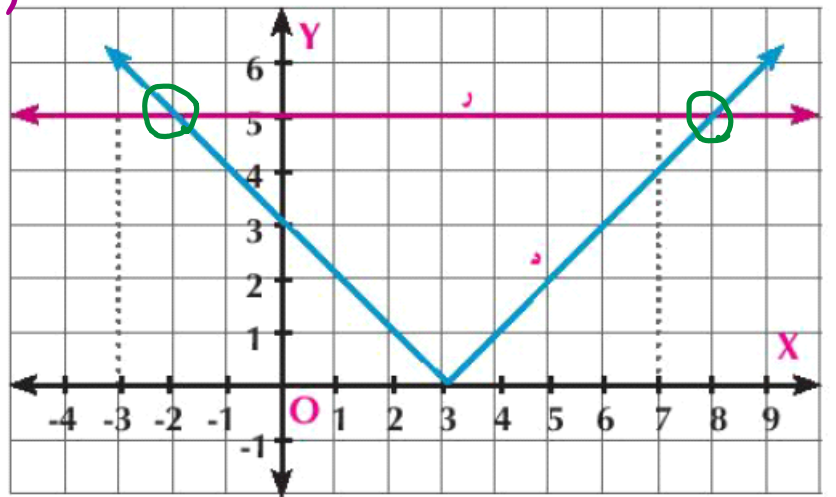
$g(x) = 5$ → const. f

algebra

$$f(x) = \begin{cases} x - 3, & x \geq 3 \\ -x + 3, & x < 3 \end{cases}$$

when $x - 3 = 5 \Rightarrow x = 8$

$-x + 3 = 5 \Rightarrow x = -2$



S.S = $\{(8, 5), (-2, 5)\}$

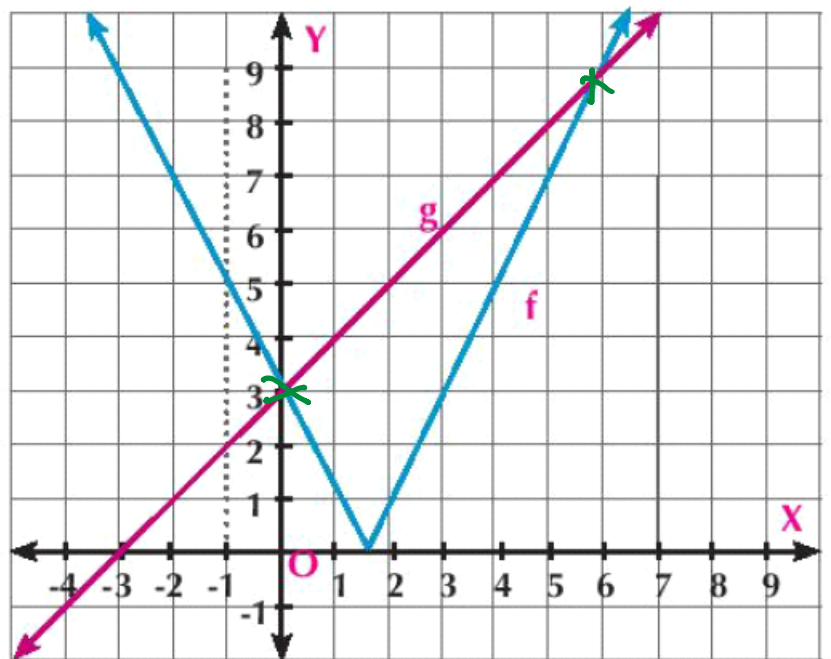
$x \rightarrow$ S.S = $\{-2, 8\}$

graph.

2 Solve the equation: $|2x - 3| = x + 3$ graphically and algebraically.

$f(x) = 2x - 3$ Linear

x	-3	-2	-1	0	1	2	3	4	5
y	9	7	5	3	1	1	3	5	7



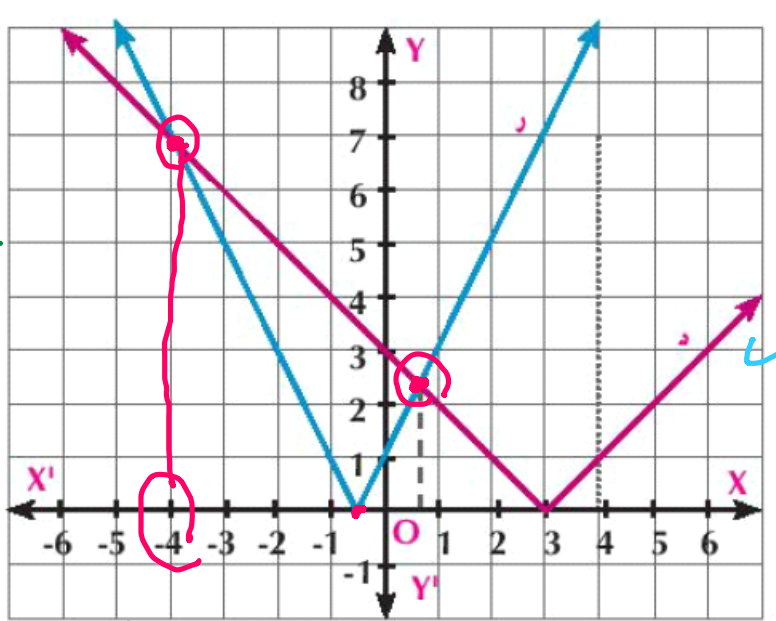
Solving absolute Value Equations and Inequalities_2

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3 Solve the equation $|x - 3| = |2x + 1|$ graphically.

$f(x) = |x - 3|$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	9	8	7	6	5	4	3	2	1	0	1	2



vertex

vertex:

$$f(x) = a|x - h| + k$$

(h, k)

S.S. = $\{-4, \frac{1}{2}\}$

$g(x) = |2x + 1|$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	11	9	7	5	3	1	1	3	5	7	9	11

$$g(x) = 2|x + \frac{1}{2}|$$

$$g(x) = a|x - h| + k$$

(h, k)

$(-\frac{1}{2}, 0)$

4 Find the solution set for each algebraically:

a $|x + 7| = |x - 5|$

$\therefore |x + 7| = |x - 5|$

$x + 7 = \pm(x - 5)$

~~$x + 7 = x - 5$~~

$7 = -5$

refused

$x + 7 = -x + 5$

$\frac{2x}{2} = 5 - 7 = \frac{-2}{2}$

$x = -1$

S.S. = $\{-1\}$

b $\sqrt{x^2 + 6x + 9} = 9 - 2x$

① $x^2 + 6x + 9 = 81 - 36x + 4x^2$

$4x^2 - x^2 - 36x - 6x + 81 - 9 = 0$

$\frac{3x^2 - 42x + 72}{3} = 0$

$x^2 - 14x + 24 = 0$

$(x - 8)(x - 3) = 0$

$x = 8 \quad x = 3$

$\sqrt{x^2 + 6x + 9} = 9 - 2x$

$\sqrt{(x + 3)^2} = 9 - 2x$

$x + 3 = \pm(9 - 2x)$

$x + 3 = 9 - 2x$

$3x = 6$

$x = 2$

$$\begin{array}{l} \boxed{\sqrt{(x+3)^2}} = 9-2x \\ |x+3| = 9-2x \end{array} \quad \left\{ \begin{array}{l} x=6 \\ x=2 \end{array} \right\} \quad \boxed{x=+6} \quad \checkmark$$

Solving absolute Value Equations and Inequalities_3

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7 Find the solution set of each of the following inequalities in form of an interval:

a $|x - 3| < 4$

b $\sqrt{x^2 - 2x + 1} > 4$

c $\frac{1}{|2x - 3|} > 2$

$|x - 3| < 4 \implies -4 + 3 < x - 3 + 3 < 4 + 3$

S.S = $]-1, 7[$

$-1 < x < 7$

b $\sqrt{x^2 - 2x + 1} > 4$

$\sqrt{(x - 1)^2} > 4$

$|x - 1| > 4$

$x - 1 > 4$

$x > 5$

$]5, \infty[$

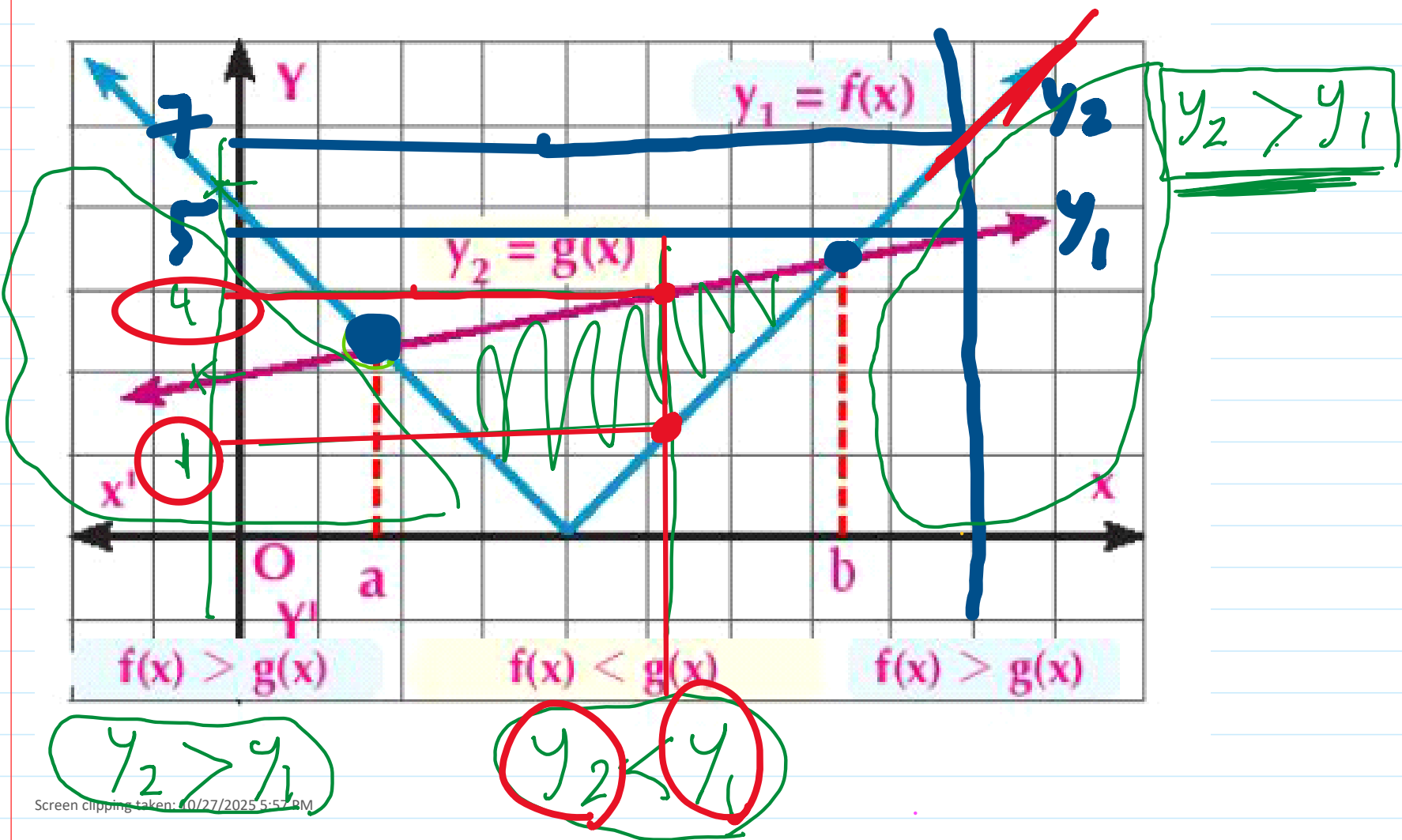
$x - 1 \leq -4$

$x \leq -3$

$]-\infty, -3]$

S.S = $]-\infty, -3] \cup]5, \infty[$

S.S = $\mathbb{R} -]-3, 5]$



Solving absolute Value Equations and Inequalities_4

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$$f(x) = |x-2|$$

$$(2,0)$$

$$(3,3)$$

Find graphically in \mathbb{R} the solution set of each of the following inequalities:

(1) $|x+3| < 2$

(2) $|2x-8| \leq 6$

$$g(x) = -|x-3| + 3$$

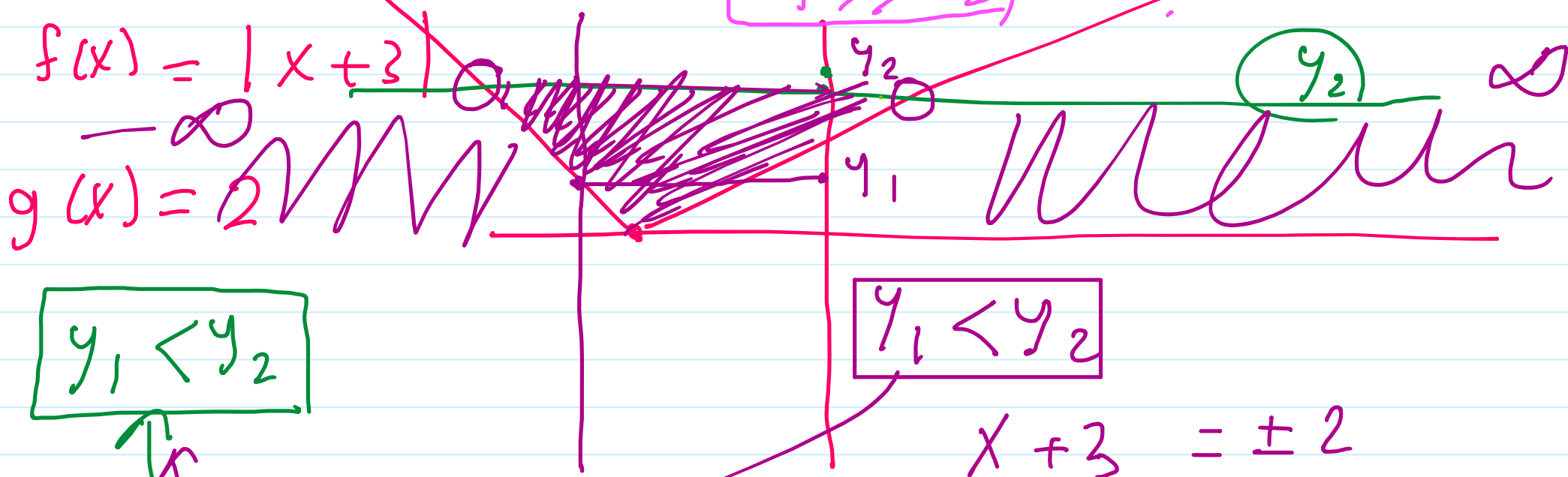
(3) $|x-3| > \frac{1}{2}x$

(4) $|x-2| \geq 3 - |x-3|$

$$y_1 > y_2$$

$$y_1$$

$$y_2$$



$$y_1 < y_2$$

$$y_1 < y_2$$

$$x+3 = \pm 2$$

$$x+3 = 2$$

$$x+3 = -2$$

$$x = -1$$

$$x = -5$$

$$S.S =]-5, -1[$$

$|x-3| > \frac{1}{2}x$

$(3,0)$

$\cup \cap$

$\vee \wedge$

