

figure (1)

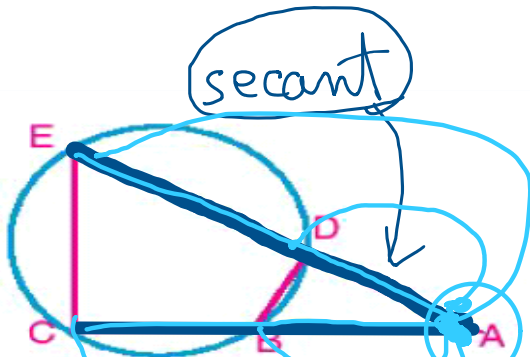


figure (2)

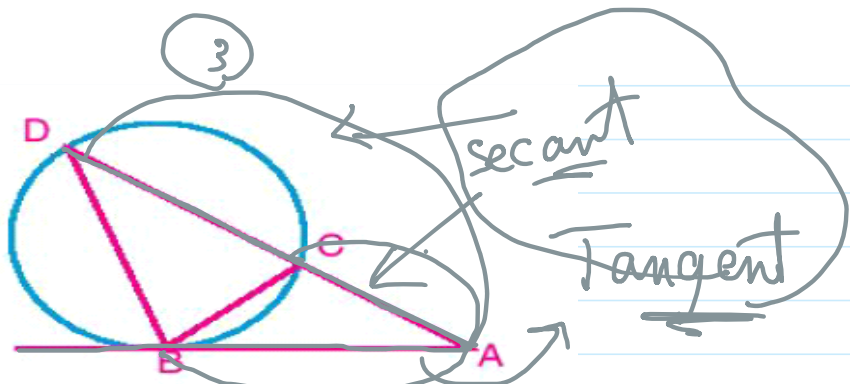


figure (3)

➤ In figure (1): Is there a relation between $EA \times EB$ and $EC \times ED$?

chords

➤ In figure (2): Is there a relation between $AE \times AD$ and $AC \times AB$?

secants

➤ In figure (3): Is there a relation between $AD \times AC$ and $(AB)^2$?

tangent + secant

$$EA \times EB = EC \times ED \Rightarrow$$

1 In the figure opposite: $\overline{AB} \cap \overline{CD} = \{E\}$, 2 chord

if $\frac{EA}{EB} = \frac{4}{3}$, $EC = 9\text{cm}$ and $ED = 4\text{cm}$

Find the length \overline{EB}

$$EA = 4x$$

$$EB = 3x$$

$$DE \times EC = BE \times EA$$

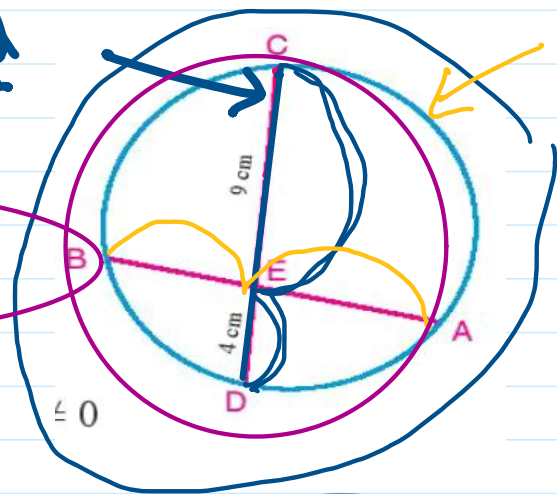
$$4 \times 9 = 4x \times 3x$$

$$\frac{36}{12} = \frac{12x^2}{12}$$

$$x^2 = 3$$

$$x = \sqrt{3}$$

$$BE = 3\sqrt{3} \text{ cm}$$



2 In the figure opposite: $\overline{AB} \cap \overline{CD} = \{E\}$, $AB = 5\text{cm}$,

$CD = 9\text{cm}$, $ED = 3\text{cm}$. Find the length of \overline{BE}

$$EB \times EA = ED \times EC$$

$$(x+9)(x-4) = 0$$

$$EB \times EA = 3 \times 12$$

$$x = -9$$

$$x = 4$$

$$x \times (5+x) = 36$$

refused

$$2 \times 18$$

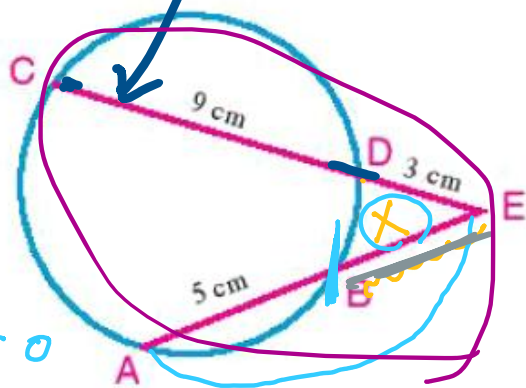
$$5x + x^2 - 36 = 0$$

$$3 \times 12$$

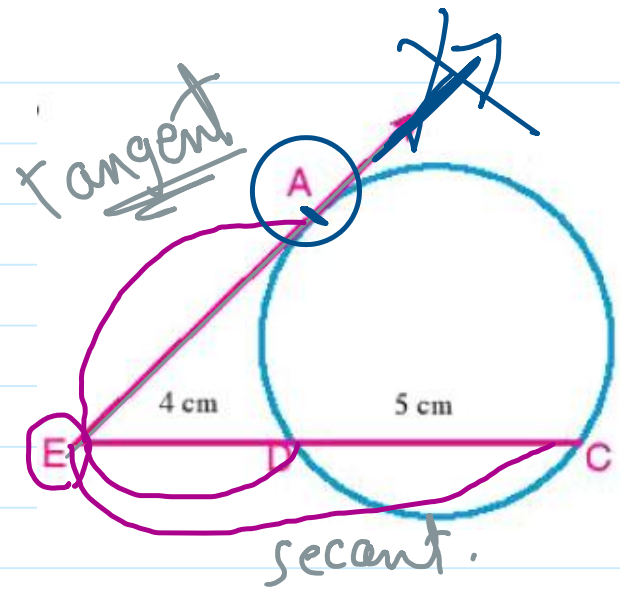
$$x^2 + 5x - 36 = 0$$

$$4 \times 9$$

$$\therefore BE = 4 \text{ cm}$$



3 In the figure opposite: \overrightarrow{EA} is a tangent to the circle, \overrightarrow{EA} intersects the circle at D and C respectively. where $ED = 4\text{cm}$, $CD = 5\text{cm}$, find the length of \overline{EA}



$$EA^2 = ED \cdot EC$$

$$= 4 \cdot 9 = 36$$

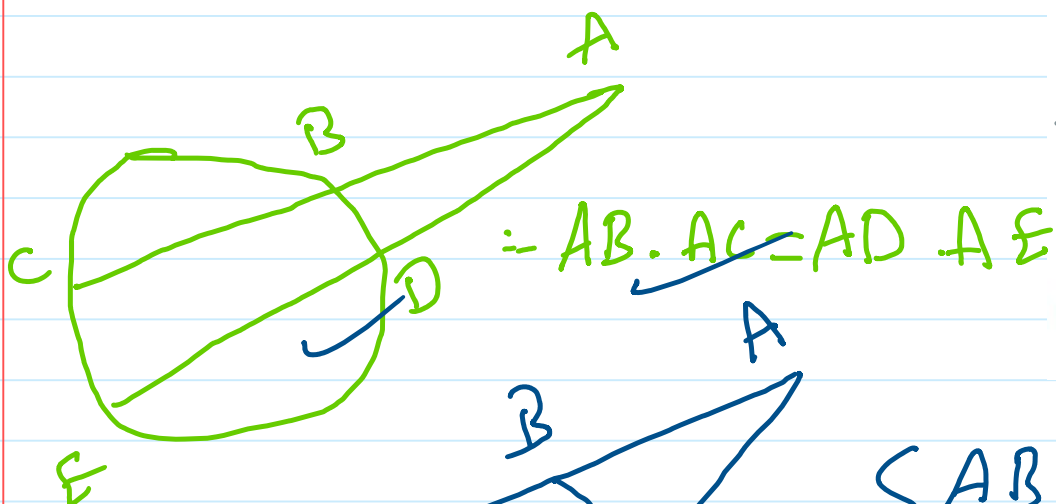
$$\sqrt{EA^2} = \sqrt{36}$$

$$EA = 6$$

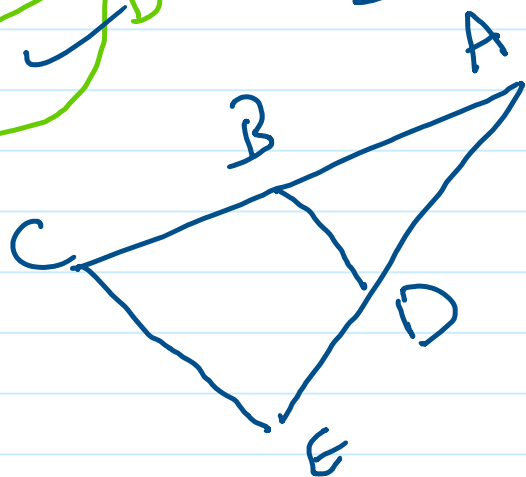
$$\therefore EA = 6\text{cm}$$

Converse of the well known problem

If the two lines containing the two segments \overline{AB} and \overline{CD} intersect at a point E (A, B, C, D, and E are distinct points) and $EA \times EB = EC \times ED$ then: the points A, B, C and D lie on a circle.



$$= AB \cdot AC = AD \cdot AE$$

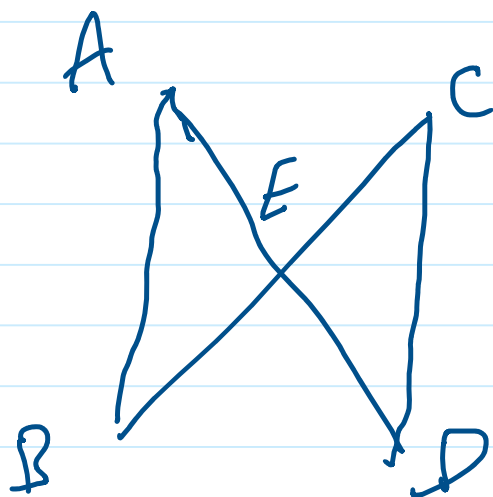


$$\begin{cases} AB \cdot AC = 36 \\ AD \cdot AE = 36 \end{cases}$$

$$\therefore AB \cdot AC = AD \cdot AE$$

\therefore we have cir. pass through points

$$\boxed{B, C, D, E}$$

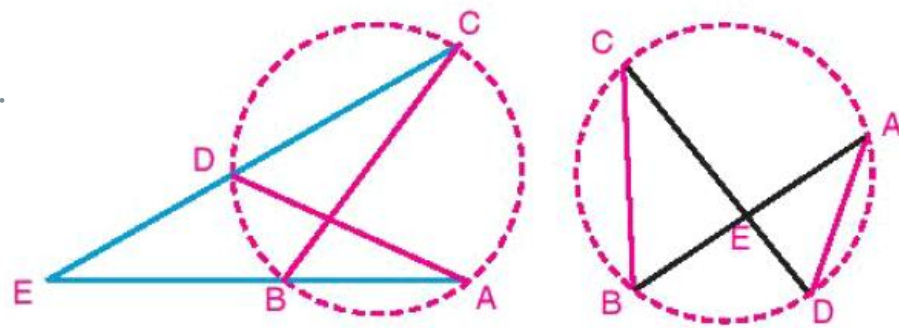


$$CE \cdot EB = 72$$

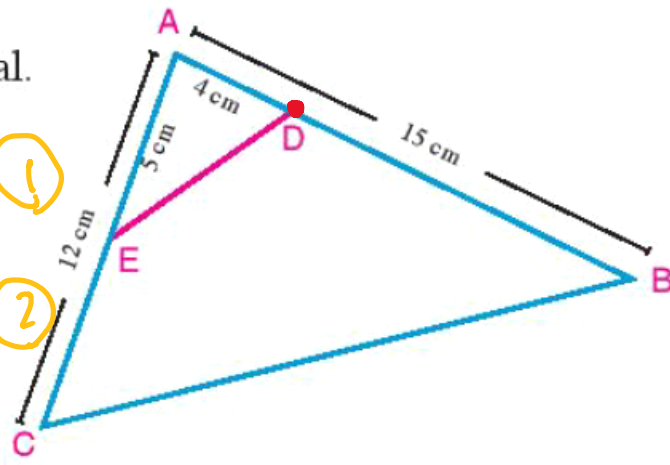
$$DE \cdot EA = 72$$

$$\therefore CE \cdot EB = DE \cdot EA$$

\therefore we have cir. pass through points A, B, C & D



4 ABC is a triangle in which $AB = 15\text{cm}$, $AC = 12\text{cm}$. $D \in \overline{AB}$ where $AD = 4\text{cm}$, $E \in \overline{AC}$ where $AE = 5\text{cm}$.
 Prove that the figure DBCE is a cyclic quadrilateral.



$\therefore AE \cdot AC = 5 \times 12 = 60 \rightarrow \textcircled{1}$

$\therefore AD \cdot AB = 4 \times 15 = 60 \rightarrow \textcircled{2}$

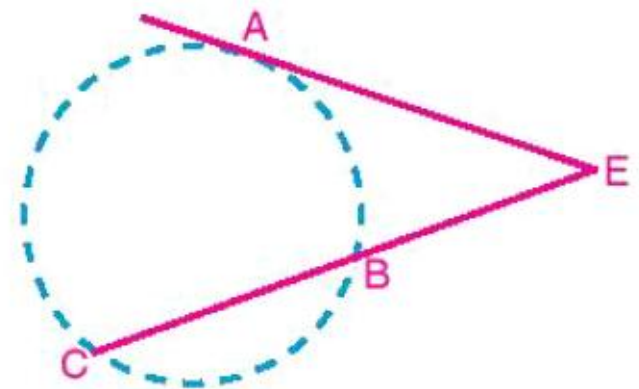
From $\textcircled{1}$ & $\textcircled{2}$

$\therefore AE \cdot AC = AD \cdot AB$

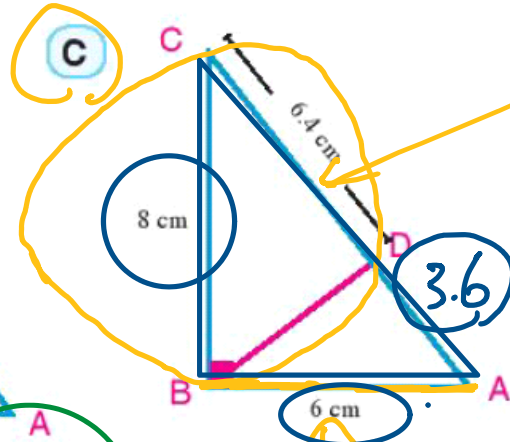
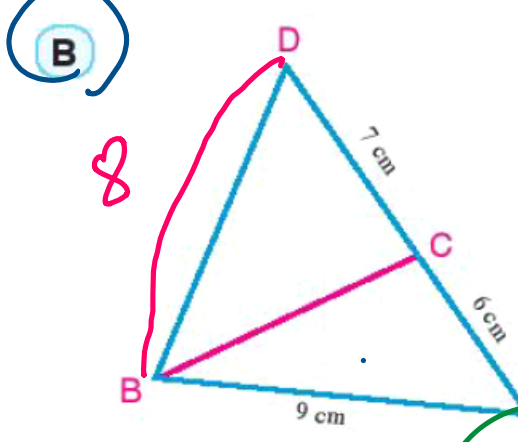
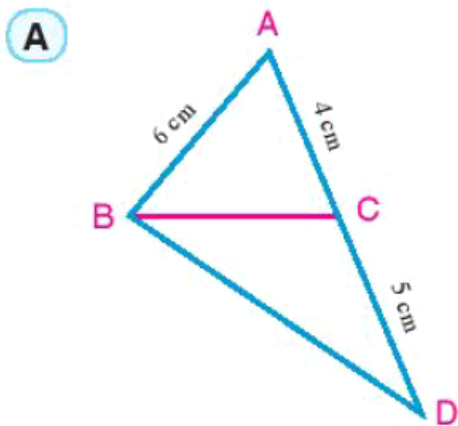
$\therefore DBCE$ is cyc. quad.

Corollary 2

If $(EA)^2 = EB \times EC$ then \overline{EA} is a tangent segment to the circle which passes through the points A, B and C.




4 In which of the following figures is \overline{AB} a tangent segment to the circle which passes through the points B, C, and D?



secant
 $AD = 10 - 6 \cdot 4 = 3.6 \text{ cm}$

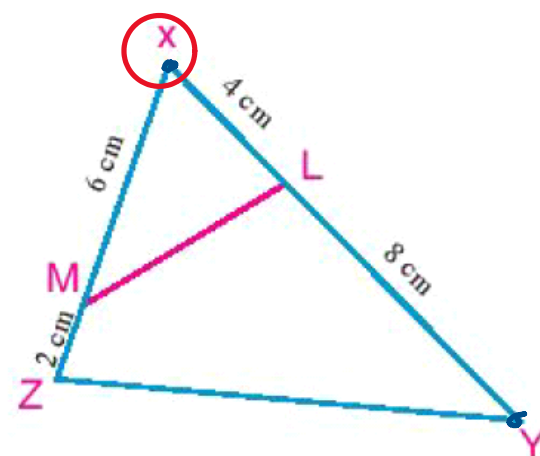
$\therefore AC \cdot AD = 6 \times 13 = 78$
 $DB^2 = 8^2 = 64$
 $AB^2 = 9^2 = 81$
 $DB^2 \neq AC \cdot AD$
 $AB^2 \neq AC \cdot AD$

AB tangent
 $\therefore AB^2 = 6^2 = 36 \rightarrow \textcircled{1}$
 $AC = \sqrt{6^2 + 8^2} = 10 \rightarrow \textcircled{2}$
 $AD \cdot AC = 3.6 \times 10 = 36$
 from $\textcircled{1}$ & $\textcircled{2} \therefore AB^2 = AD \cdot AC$
 $\therefore AB$ is tangent.

|  |

| $\therefore AB$ is tangent.

6 In the figure opposite: $L \in \overline{XY}$ where $XL = 4\text{cm}$, $YL = 8\text{cm}$, $M \in \overline{XZ}$ where $XM = 6\text{cm}$, $ZM = 2\text{cm}$
 Prove that:



A $\triangle XLM \sim \triangle XZY$

B LYZM is a cyclic quadrilateral.

$$\therefore \frac{XL}{XZ} = \frac{4}{8} = \frac{1}{2} \longrightarrow \textcircled{1}$$

$$\therefore \frac{XM}{XY} = \frac{6}{12} = \frac{1}{2} \longrightarrow \textcircled{2}$$

from $\textcircled{1}$ & $\textcircled{2}$ $\therefore \frac{XL}{XZ} = \frac{XM}{XY} \therefore \triangle XLM \sim \triangle XZY$

~~$ZM \cdot ZX = 2 \cdot 8 = 16$~~
 ~~$YL \cdot YX = 8 \cdot 12 = 96$~~

$XM \cdot XZ = 6 \cdot 8 = 48$
 $XL \cdot XY = 4 \cdot 12 = 48$

$\therefore XM \cdot XZ = XL \cdot XY \therefore LYZM$ is cyc. quad.

8) $\triangle ABC$ is a triangle. $D \in \overline{BC}$ where $DB = 5\text{cm}$ and $DC = 4\text{cm}$. If $AC = 6\text{cm}$. Prove that:

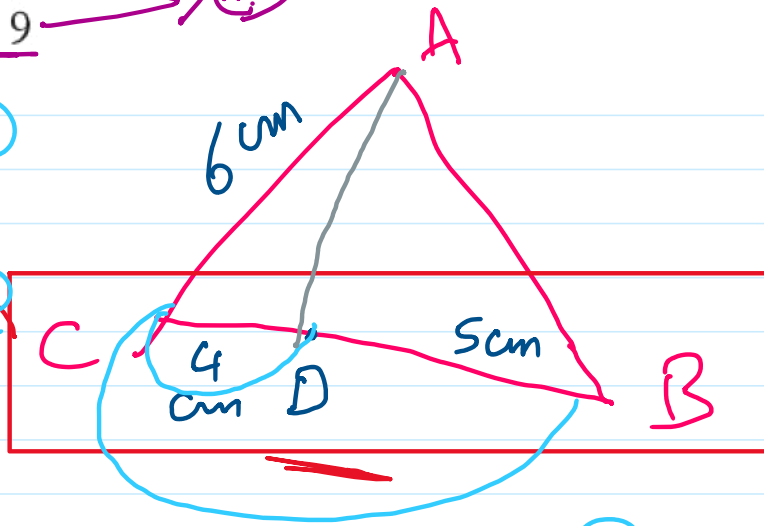
~~A~~ \overline{AC} is a tangent segment to the circle passing through the points A, B and D.

~~B~~ $\triangle ACD \sim \triangle BCA$

~~C~~ Area of $(\triangle ABD)$: area of $(\triangle ABC) = 4:9$

$AC^2 = 6^2 = 36 \rightarrow 1$

$CD \cdot CB = 4 \cdot 9 = 36 \rightarrow 2$



$\therefore AC^2 = CD \cdot CB$

$\therefore \overline{AC}$ is a tangent segment $\dots \dots \dots \neq A$

$\therefore \frac{AC}{BC} = \frac{6}{9} = \frac{2}{3} \rightarrow 3$

$\frac{CD}{CA} = \frac{4}{6} = \frac{2}{3} \rightarrow 4$

$\therefore \frac{AC}{BC} = \frac{CD}{CA} = \frac{2}{3}$

$\therefore \triangle ACD \sim \triangle BCA$ ~~B~~

$\left(\frac{CD}{CA}\right)^2 = \frac{A \cdot \triangle ABD}{A \cdot \triangle ABC} = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \neq C$