

Limits of trig. Functions_1

Monday, November 3, 2025 11:44 PM

Theorem

If x is the measure of an angle in radian then :

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 1} \frac{\sin x}{x} = 1$$

$$\frac{\sin x}{x} = 1$$

Corollary 1:

$$\lim_{x \rightarrow 0} \frac{\sin ax}{x} = a, \quad \lim_{x \rightarrow 0} \frac{\tan ax}{x} = a$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{3x} = \frac{2}{3}$$

Corollary 2

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$$

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Example:

$$\lim_{x \rightarrow 0} \frac{\sin 5x \cos 2x}{x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \cdot \lim_{x \rightarrow 0} \cos 2x = 5 \times \cos 0 = 5 \times 1 = 5$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\tan x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \cdot \frac{x}{\tan x} = 0 \times 1 = 0$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \times \frac{1 + \cos x}{1 + \cos x} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2 (1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 (1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x} = \frac{1}{2} \times 1 = \frac{1}{2}$$

$$x^2 - y^2 = (x - y)(x + y)$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin 2x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \cdot \lim_{x \rightarrow 0} \frac{x}{\sin 2x} = 0 \times \frac{1}{2} = 0$$

Limits of trig. Functions_2

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$$x^2 - x =$$

Find the following limits :

a $\lim_{x \rightarrow 0} \frac{x^2 - x + \sin x}{2x}$

b $\lim_{x \rightarrow 0} \frac{x + x \cos x}{\sin x \cos x}$

c $\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x}$

d $\lim_{x \rightarrow 0} \frac{\sin^2 3x}{5x^2}$

a $\lim_{x \rightarrow 0} \frac{x^2}{2x} - \frac{x}{2x} - \frac{\sin x}{2x}$

$$\lim_{x \rightarrow 0} \left(\frac{x}{2} - \frac{1}{2} - \frac{\sin x}{2x} \right)$$

$$\lim_{x \rightarrow 0} \frac{x}{2} - \lim_{x \rightarrow 0} \frac{1}{2} - \lim_{x \rightarrow 0} \frac{\sin x}{2x}$$

$$0 - \frac{1}{2} - \frac{1}{2} = \boxed{-1}$$

b $\lim_{x \rightarrow 0} \frac{x(1 + \cos x)}{\sin x \cos x} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \lim_{x \rightarrow 0} \frac{1 + \cos x}{\cos x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{x}} = \frac{1}{1} = 1$$

$$\frac{1+1}{1} = 2$$

c $\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x} = 1$

Let $y = (1 - \cos x)$

$$\lim_{x \rightarrow 0} \frac{\sin y}{y} = 1$$