

Related angles_8

Saturday, November 1, 2025 12:08 AM

Find the S.S. of each of the following equations knowing that $\theta \in]0, 2\pi[$:

(1) $2 \cos \theta + 1 = 0$

(3) $2 \sin \theta - \sqrt{3} = 0$

(5) $2 \sin \theta + \sqrt{3} = 0$

(7) $\sqrt{3} \csc \theta = -2$

(2) $\sec \theta - \sqrt{2} = 0$

(4) $\cos \theta + 1 = 0$

(6) $\tan \theta + 1 = 0$

(8) $\sin^2 \theta = \frac{1}{4}$

$2 \cos \theta + 1 = 0$

S.S. = ? $\theta \in]0, 2\pi[$

$\frac{2 \cos \theta}{2} = -\frac{1}{2}$

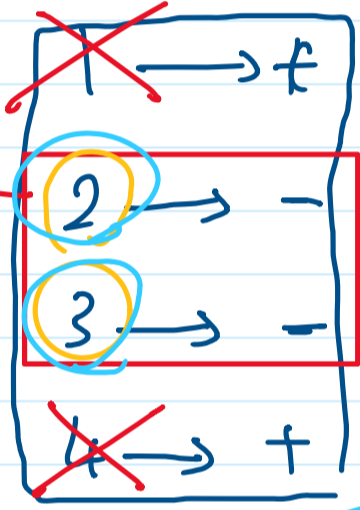
$\cos \theta = -\frac{1}{2}$

1, 2, 3, 4

$\theta = -60$

4th

$(180 - \theta)$
 $(180 + \theta)$



$\cos \theta = -\frac{1}{2}$

$\cos(180 - 60) = \cos 120$

$\theta = 120$

$\cos(180 + 60) = \cos 240$

$\theta = 240$

$(180 - 120) = 60$

$(180 + \theta) = 180 + 60 = 240$

$\therefore \text{S.S.} = \{120, 240\}$

$\frac{\sqrt{3} \csc \theta}{\sqrt{3}} = -\frac{2}{\sqrt{3}}$

$\csc \theta = -\frac{2}{\sqrt{3}}$

~~$\frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$~~

2nd $\rightarrow 180 + 60 = 240$

4th $\rightarrow 360 - 60 = 300$

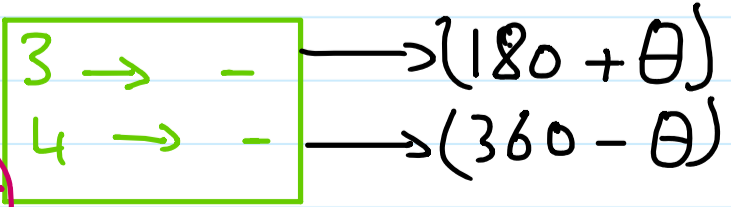
$\sin \theta = -\frac{\sqrt{3}}{2}$

$\theta = -60$
4th

1 \rightarrow +
2 \rightarrow +

S.S. = $\{240, 300\}$

$\sin 240 = -\frac{\sqrt{3}}{2}$
 $\sin 300 = -\frac{\sqrt{3}}{2}$



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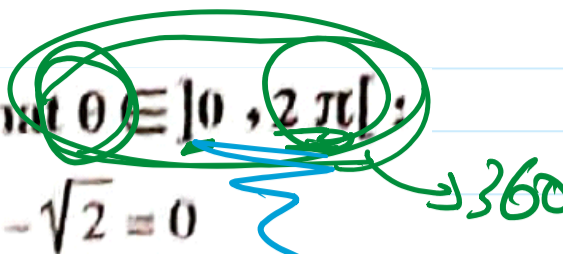
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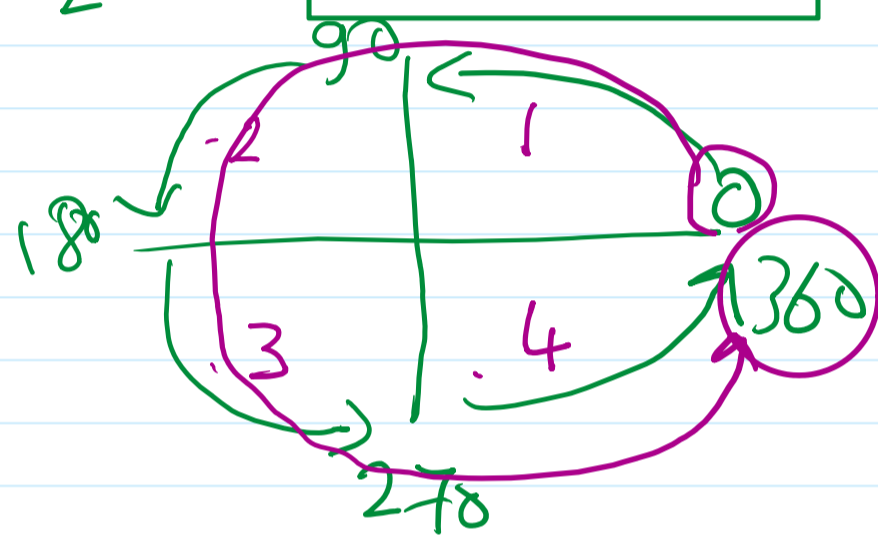
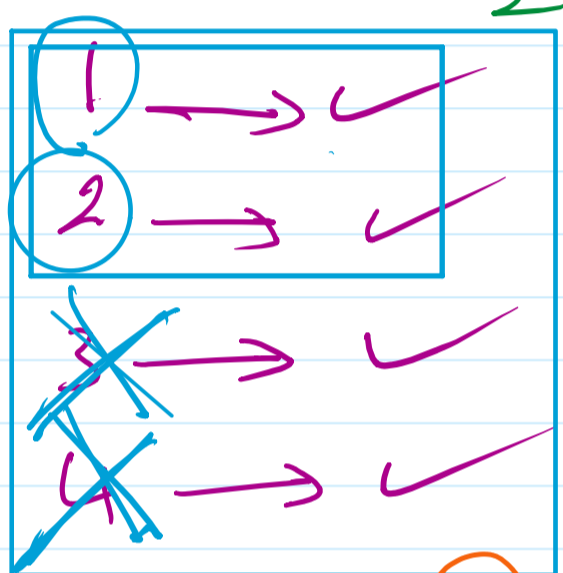
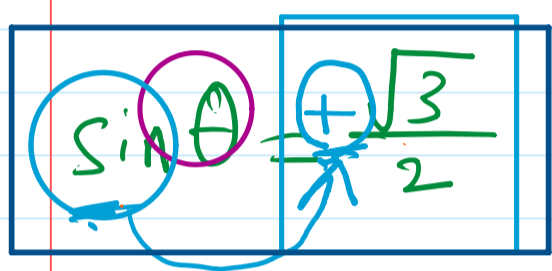
(8) $\sin^2 \theta = \frac{1}{4}$



(3) $2 \sin \theta - \sqrt{3} = 0$

$\frac{2 \sin \theta}{2} = \frac{\sqrt{3}}{2}$

$\sin \theta = \frac{\sqrt{3}}{2}$



S.S = { 60, 120 }

①

$\theta = 60$

② $180 - \theta$

$180 - 60 = 120$

③ $180 + \theta$

$= 180 + 60 = 240$

$\sin 60 = \frac{\sqrt{3}}{2}$ ✓

$\sin 120 = \frac{\sqrt{3}}{2}$ ✓

$\sin 240 = -\frac{\sqrt{3}}{2}$ ✗

$\sin 300 = -\frac{\sqrt{3}}{2}$ ✗

S.S = { 60, 120 } = 300

★ If $\sin \theta = -\frac{1}{2}$, θ is the smallest positive measure, then $\theta = \dots\dots\dots^\circ$

(a) 30

(b) 150

(c) 210

(d) 330

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General solution of trigonometric equations in the form

[sin(α) = cos(β), sec(α) = csc(β), tan(α) = cot(β)]

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you have studied before that, if α and β are the measures of two complementary angles (their sum equals 90°) then sin α = cos β, sec α = csc β, tan α = cot β, hence α + β = 90° where α and β are two acute angles, If sin θ = cos 15° then what are expected values of θ?

1- If sin α = cos β (where α, β are the measures of two complementary angles) then:

➤ sin α = sin(π/2 - β) **hence:** α = π/2 - β **i.e.** α + β = π/2

➤ sin α = sin(π/2 + β) **hence:** α = π/2 + β **i.e.** α - β = π/2

α + β = 90

Add 2πn (where n ∈ Z) to the angle π/2 then:

when sin α = cos β then α ± β = π/2 + 2πn (n ∈ Z), similarly:

when csc α = sec β then α ± β = π/2 + 2πn (n ∈ Z), α ≠ nπ, β ≠ (2n + 1)π/2

cos 30 = sin 60
√3/2 = √3/2

sin 20 = cos 70
0.34 = 0.34

2- If tan α = cot β (where α, β are the measure of two complementary angles) then:

➤ tan α = tan(π/2 - β) **hence:** α = π/2 - β **i.e.** α + β = π/2

➤ tan α = tan(3π/2 - β) **hence:** α = 3π/2 - β **i.e.** α + β = 3π/2

Add 2πn (where n ∈ Z) to the two angles π/2 and 3π/2 then:

when tan α = cot β then α + β = π/2 + πn (where n ∈ Z), α ≠ (2n + 1)π/2, β ≠ nπ

sin 30 = cos 220
0.5 = -0.5

90 + 360 × 1 = 450
90 + 360 × 2 = 810
90 + 360 × 3 = 1170

π/2 + 2πn

sin α = cos β
csc α = sec β

α ± β = π/2 + 2πn

$$\frac{\pi}{2} + 2\pi n$$

$$\alpha \pm \beta = \frac{\pi}{2} + 2\pi n$$

without

$$\sin(90 + \theta) = \cancel{\sin \theta} \cos(-120)$$

$$-\cos 120$$

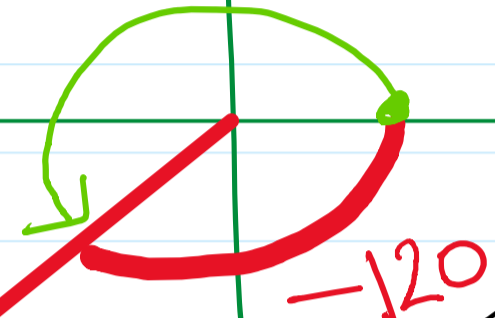
$$\begin{matrix} 360 - \theta \\ 180 + \theta \end{matrix}$$

$$-\frac{1}{2}$$

$$\begin{matrix} 90 - \theta \\ 90 + \theta \\ 270 + \theta \\ 270 - \theta \end{matrix}$$

-180

$$-\frac{1}{2}$$



$$\cos(-120)$$

$$+ \cos(120)$$

~~$$(\cos 180 + 60)$$~~

$$\begin{aligned} &\cos(-120 + 360) \\ &-\frac{1}{2} \cos(240) \\ &-90 \cos(180 + 60) \\ &= -\cos 60 \\ &-\frac{1}{2} \end{aligned}$$

$$\begin{aligned} &\cos(-120) \\ &+ \cos(120) \\ &-\frac{1}{2} \end{aligned}$$

If $\sin \theta = -\frac{1}{2}$, θ is the smallest positive measure, then $\theta = \dots\dots\dots^\circ$

(a) 30

(b) 150

(c) 210

(d) 330

$\sin \theta = -\frac{1}{2} \longrightarrow \theta = 30^\circ$

