

Limits of trig. Functions_3

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Find the following limits :

a $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x}$

b $\lim_{x \rightarrow 0} \frac{\sin(x-1)}{x^2+x-2}$

c $\lim_{x \rightarrow 0} \frac{\sin^2 x}{3x^2}$

d $\lim_{x \rightarrow 1} \frac{x \sin 2x + \sin^2 2x}{\tan^2 3x + x^2}$

a $\lim_{x \rightarrow 0} \frac{\sin 3x}{5x^3 - 4x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{x(5x^2 - 4)}$
 $= \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{5x^2 - 4} = 3 \times \frac{1}{-4} = \boxed{-\frac{3}{4}}$

d $\lim_{x \rightarrow 0} \frac{x \sin 2x + \sin^2 2x}{\tan^2 2x + x^2} = \lim_{x \rightarrow 0} \frac{\sin 2x (x + \sin 2x)}{\tan^2 2x + x^2}$
 $\lim_{x \rightarrow 0} \frac{\left(\frac{\sin 2x}{x} + \frac{\sin^2 2x}{x^2} \right)}{\left(\frac{\tan^2 2x}{x^2} + 1 \right)} = \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x} + \frac{\sin^2 2x}{x^2} \right) / \lim_{x \rightarrow 0} \left(\frac{\tan^2 2x}{x^2} + 1 \right)$
 $= \frac{2 + 2^2}{2^2 + 1} = \frac{6}{5}$
 ~~$\frac{4}{3}$~~
 $\frac{\sin 2x}{x} \times \frac{\sin 2x}{x} = 2 \times 2 = \boxed{4}$

Limits of trig. Functions_4

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- (9) $\lim_{x \rightarrow 0} \frac{x^2 + \tan^2 2x}{2x^2 + \sin^2 3x}$ "5/11"
- (10) $\lim_{x \rightarrow 0} \frac{x \tan x}{4x^2 - \sin^2 3x}$ "-1/5"
- (11) $\lim_{x \rightarrow 0} \frac{2x^3 + x \sin 5x}{x^2 - \tan 3x^2}$ "-5/2"
- (12) $\lim_{x \rightarrow 0} \frac{2x + 5 \tan x + 3 \sin x}{\sin 2x \cos 2x}$ "5"
- (13) $\lim_{x \rightarrow 0} \frac{x \sin 2x + \sin^2 2x}{\tan^2 3x + x^2}$ "3/5"
- (14) $\lim_{x \rightarrow 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$ "28"
- (15) $\lim_{x \rightarrow 0} \frac{\sin 5x^2 + \sin^3 5x}{2x^3}$ "65"
- (16) $\lim_{x \rightarrow 0} \left(\frac{2x^2 + \sin 3x}{2x^2 + \tan 6x} \right)^4$ "1/16"
- (17) $\lim_{x \rightarrow 0} \frac{\tan^2 x + \tan^2 3x + \tan^2 5x}{x^2}$ "35"
- (18) $\lim_{x \rightarrow 0} \frac{\sin x \sin^2 2x \sin^3 4x}{x^6}$ "256"

(11) $\lim_{x \rightarrow 0} \frac{x^2(2x + \frac{\sin 5x}{x})}{x^2(1 - \frac{\tan 3x^2}{x^2})}$

$= \frac{5}{1-3} = \frac{5}{-2} = -\frac{5}{2}$

(15) $\lim_{x \rightarrow 0} \frac{\frac{\sin 5x^3}{x^3} + \frac{\sin^3 5x}{x^3}}{2}$

$= \lim_{x \rightarrow 0} \left(\frac{\sin 5x^3}{x^3} + \frac{\sin^3 5x}{x^3} \right) = \frac{5}{2} + \left(\frac{5}{2} \right)^3$

(15) $\lim_{x \rightarrow 0} \left(\frac{\sin 5x^3}{2x^3} + \frac{\sin^3 5x}{2x^3} \right) = \frac{5}{2} + \left(\frac{5}{2} \right)^3$

Handwritten calculations for problem 11:

$\frac{\tan^2 3x}{x^2} = 3^2 = 9$

$\frac{\tan^2 3x^2}{x^2} = 3 = 3$

$\frac{\tan 3y}{y} = 3$

Final result: $5 + 5^3$

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(9) $\lim_{x \rightarrow 0} \frac{x^2 + \tan^2 2x}{2x^2 + \sin^2 3x}$ « $\frac{5}{11}$ »	(10) $\lim_{x \rightarrow 0} \frac{x \tan x}{4x^2 - \sin^2 3x}$ « $-\frac{1}{5}$ »
(11) $\lim_{x \rightarrow 0} \frac{2x^3 + x \sin 5x}{x^2 - \tan 3x^2}$ « $-\frac{5}{2}$ »	(12) $\lim_{x \rightarrow 0} \frac{2x + 5 \tan x + 3 \sin x}{\sin 2x \cos 2x}$ « 5 »
(13) $\lim_{x \rightarrow 0} \frac{x \sin 2x + \sin^2 2x}{\tan^2 3x + x^2}$ « $\frac{3}{5}$ »	(14) $\lim_{x \rightarrow 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$ « 28 »
(15) $\lim_{x \rightarrow 0} \frac{\sin 5x^3 + \sin^3 5x}{2x^3}$ « 65 »	(16) $\lim_{x \rightarrow 0} \left(\frac{2x^2 + \sin 3x}{2x^2 + \tan 6x} \right)^4$ « $\frac{1}{16}$ »
(17) $\lim_{x \rightarrow 0} \frac{\tan^2 x + \tan^2 3x + \tan^2 5x}{x^2}$ « 35 »	(18) $\lim_{x \rightarrow 0} \frac{\sin x \sin^2 2x \sin^3 4x}{x^6}$ « 256 »

$$\lim_{x \rightarrow 0} \frac{2x^2 + \sin 3x}{2x^2 + \tan 6x}$$
→
()⁴

$$\lim_{x \rightarrow 0} \frac{\frac{\sin 5x}{x^3} + \frac{\sin^3 5x}{x^3}}{2} = \frac{5 + 5^3}{2}$$

$$\lim_{x \rightarrow 0} \left(\frac{\sin 5x^3}{2x^3} + \frac{\sin^3 5x}{2x^3} \right) = \frac{5}{2} + \left(\frac{5}{2} \right)^3$$