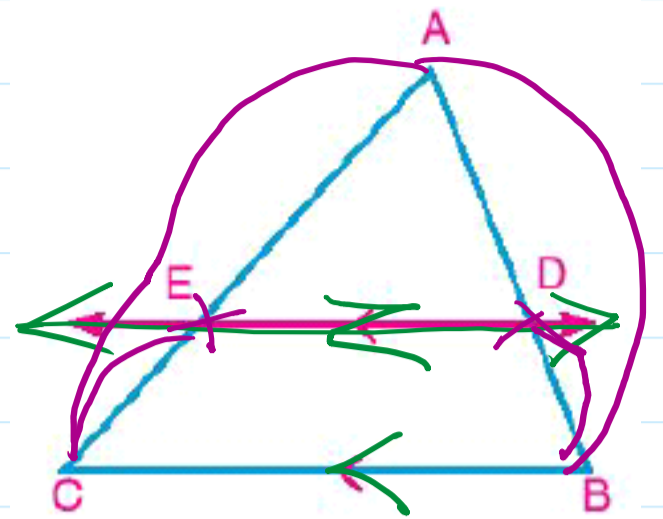


Theorem 1

If a line is drawn parallel to one side of a triangle and intersects the other two sides, then it divides them into segments whose lengths are proportional.

(Proof is not required)



i.e.: $\frac{AD}{DC} = \frac{AE}{EB}$

D & E are midpoints

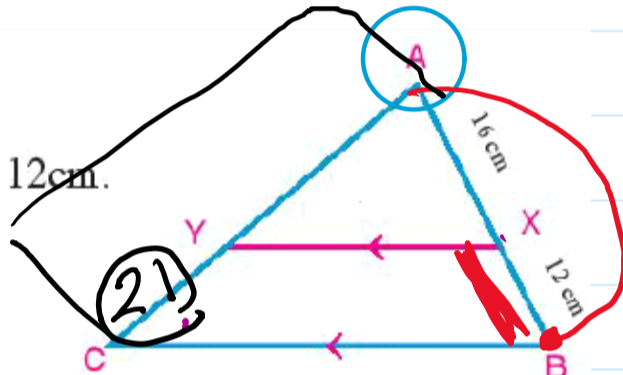
$\therefore DE \parallel BC \therefore DE = \frac{1}{2} BC$

$$\frac{AD}{DC} = \frac{AE}{EB}$$

Example

1 In the figure opposite: $\overline{XY} \parallel \overline{BC}$, $AX = 16\text{cm}$, $BX = 12\text{cm}$.

- A If $AY = 24\text{cm}$, Find YC .
- B If $CY = 21\text{cm}$, find AC .



$\therefore \overline{XY} \parallel \overline{BC}$

$$\frac{AX}{XB} = \frac{AY}{YC}$$

~~$\frac{28}{12} = \frac{AC}{21}$~~

$$28z = 12(24 + z)$$

~~$\frac{28}{12} = \frac{AC}{YC} = \frac{24+z}{z}$~~

$$28z = 288 + 12z$$

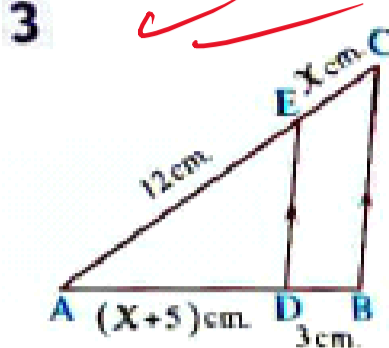
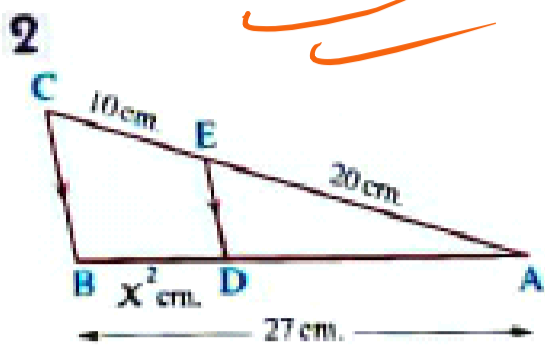
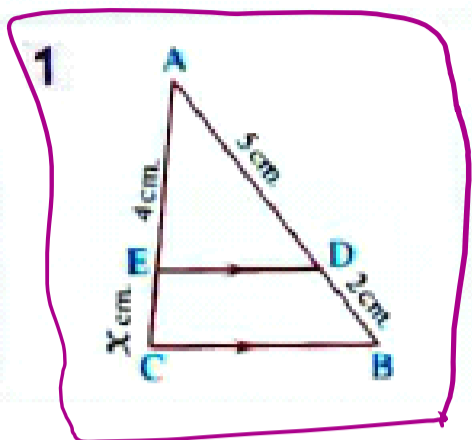
$$28z - 12z = 288$$

$$\frac{16z}{16} = \frac{288}{16}$$

$$z = 18$$

$$\therefore YC = 18$$

In each of the following figures : $DE \parallel BC$ Find the value of x



$\therefore DE \parallel BC$
 $7x = 8 + 2x$

$\therefore \frac{AB}{DB} = \frac{AC}{EC} \Rightarrow \frac{7}{2} = \frac{4+x}{x}$
 $5x = 8$
 $x = \frac{8}{5}$

$\therefore DE \parallel BC$
 $\frac{30x^2}{30} = \frac{270}{30}$

$\therefore \frac{AC}{EC} = \frac{AB}{DB} \Rightarrow \frac{30}{10} = \frac{27}{x^2}$
 $x^2 = 9$
 $x = 3$

$\therefore DE \parallel BC$

$\therefore \frac{AC}{EC} = \frac{AB}{DB} \Rightarrow \frac{12+x}{x} = \frac{x+5+3}{3}$

$3(12+x) = x(x+8)$
 $36 + 3x = x^2 + 8x$

$x^2 + 8x - 3x - 36 = 0$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$x+9=0$ | $x-4=0$
 $x=-9$ | $x=4$

$x = -9$ refused

Corollary If a straight line is drawn outside the triangle ABC parallel to one side of the triangle, say \overline{BC} intersecting \overline{AB} and \overline{AC} at D and E , respectively as shown in the figures, then : $\frac{AB}{DB} = \frac{AC}{CE}$.

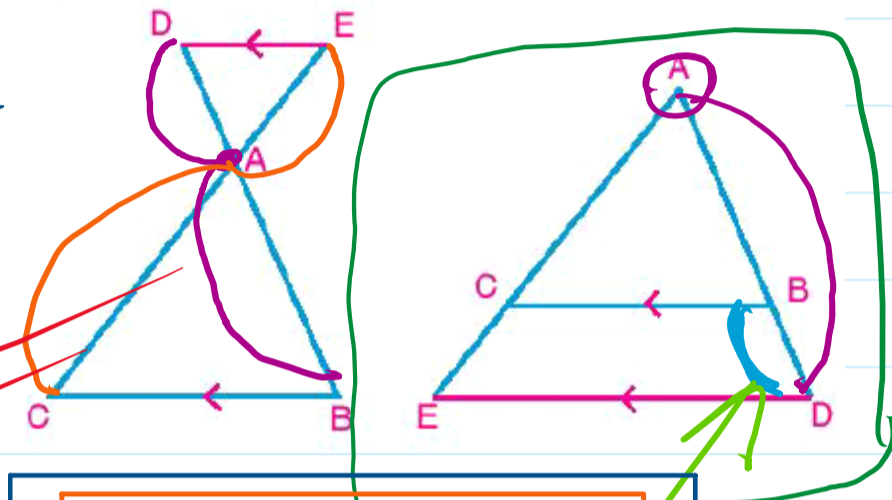
and from the properties of the proportion we can deduce that:

$$\frac{AD}{AB} = \frac{AE}{AC}, \quad \frac{AD}{DB} = \frac{AE}{CE}$$

$$\therefore \overline{DE} \parallel \overline{BC}$$

$$\frac{AB}{BD} = \frac{AC}{CE}$$

$$\frac{AD}{BD} = \frac{AE}{CE}$$



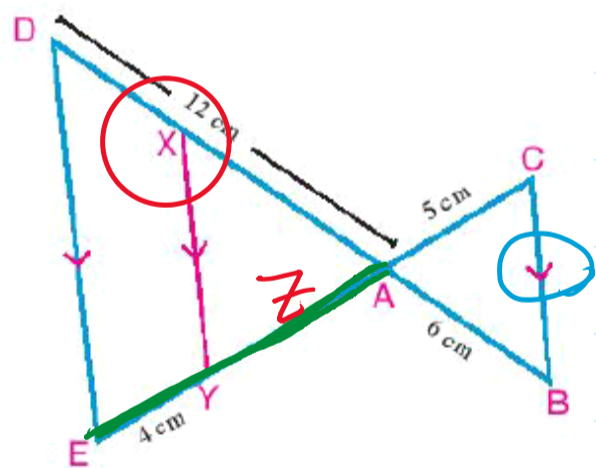
$$\therefore \overline{BC} \parallel \overline{DE}$$

$$\therefore \frac{AD}{BD} = \frac{AE}{CE}$$

given

$$\frac{AD}{AB} = \frac{AE}{AC}$$

2 In the figure opposite: $\overline{CE} \cap \overline{BD} = \{A\}$, $X \in \overline{AD}$
 $Y \in \overline{AE}$ where $\overline{XY} \parallel \overline{BC} \parallel \overline{DE}$.
 If $AB = 6\text{cm}$, $AC = 5\text{cm}$, $AE = 12\text{cm}$, $EY = 4\text{cm}$.
 Find the length of each of AX and DX .



$$\triangle AED \quad \therefore \overline{XY} \parallel \overline{ED}$$

$$\therefore \frac{AE}{YE} = \frac{AD}{XD}$$

$$\overline{CB} \parallel \overline{DE}$$

$$\therefore \frac{AB}{AD} = \frac{AC}{AE} \quad \frac{6}{12} = \frac{5}{4+7}$$

$$\frac{CA}{CE} = \frac{BA}{BD}$$

$$\therefore \overline{BC} \parallel \overline{XY}$$

$$\frac{BX}{AX} = \frac{CY}{AY}$$

$$\frac{AX}{BX} = \frac{AY}{CY}$$