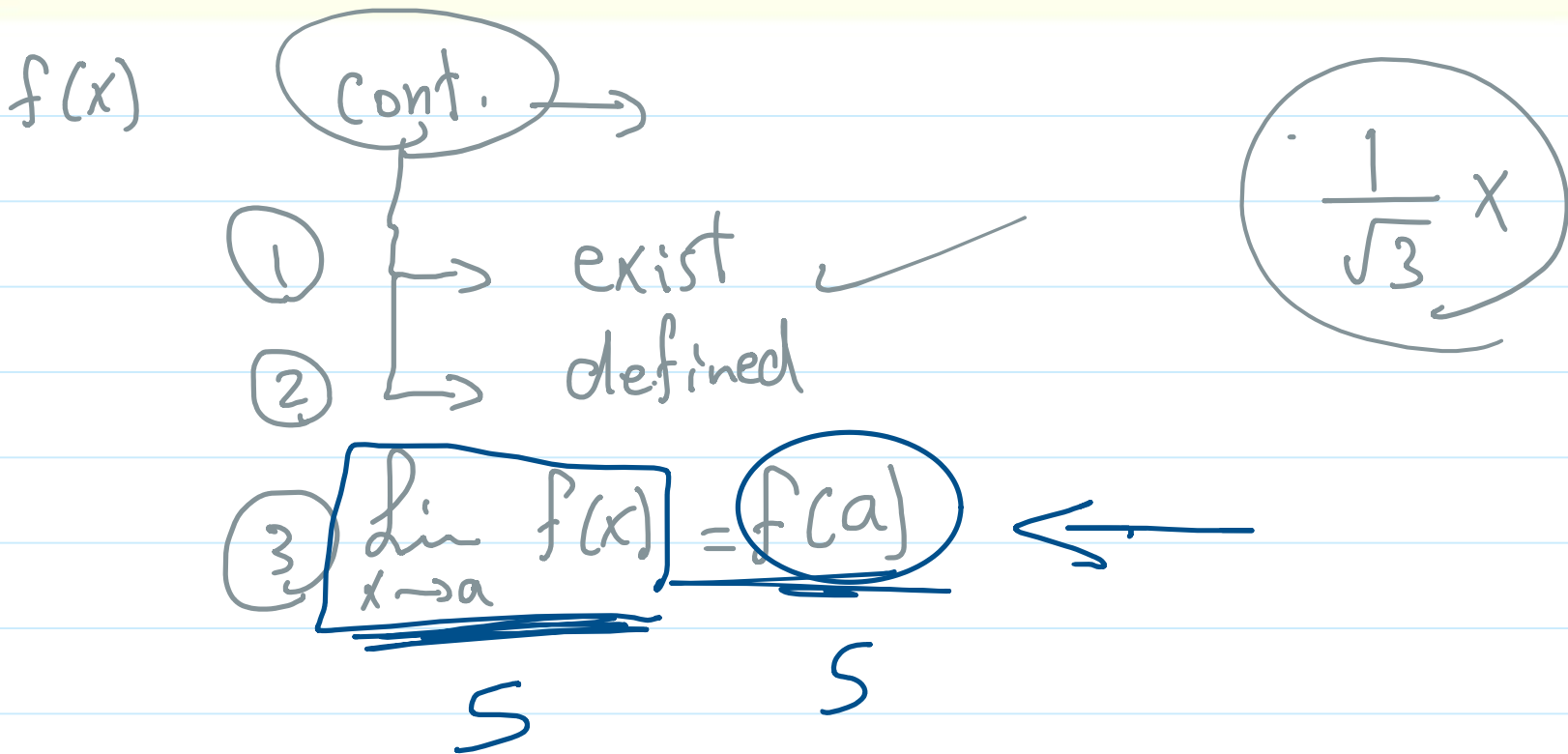


Definition 1

The function f is said to be continuous at $x = a$ if the following conditions satisfied:

- $\lim_{x \rightarrow a} f(x)$ exist
- $f(a)$ defined
- $\lim_{x \rightarrow a} f(x) = f(a)$



1 Discuss the continuity of the function f where $f(x) = \begin{cases} x \\ x+1 \end{cases}$

if $x \leq 1$
if $x > 1$ X

$f(x)$

a at $x = 0$

b at $x = 1$

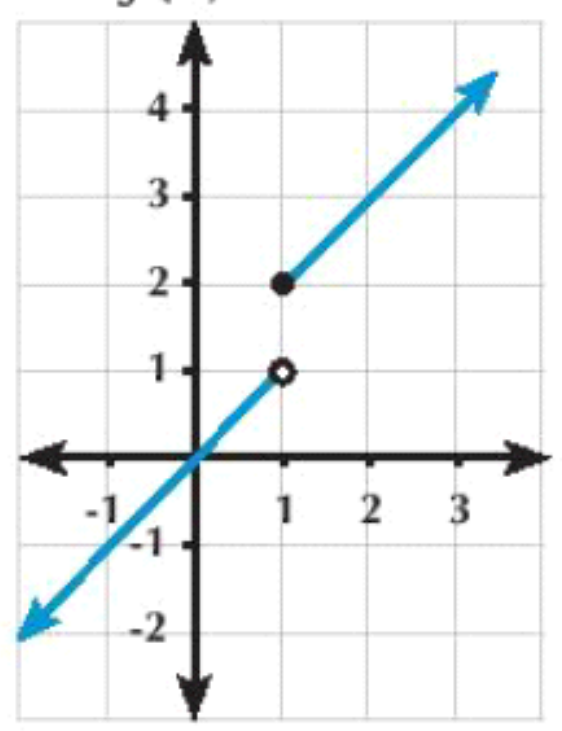
$\lim_{x \rightarrow 0^-} f(x)$ $f(0) = 0$ cont.

$\lim_{x \rightarrow 1^+} (x+1) = 2$ $f(1) = 1+1 = 2$

$\lim_{x \rightarrow 1} f(x) = f(1)$

$\lim_{x \rightarrow 1^-} x = 1$ $f(1) = 2$

$\lim_{x \rightarrow 1} f(x) = f(1)$



$\therefore f(1) \neq f(1^+)$ $\therefore f(x)$ is not exist

$\therefore f(x)$ is not cont.

Continuity_2

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$$x-3=0 \\ \boxed{x=3}$$

2) Discuss the continuity of each of the following functions at the indicated points in front of each:

a) $f(x) = \frac{x+3}{x-2}$ at $x=2, x=3$

b) $f(x) = 5 - |x-3|$ at $x=3$

$$|x-3| = \begin{cases} x-3, & x \geq 3 \\ -x+3, & x < 3 \end{cases}$$

$$f(x) = \begin{cases} 5 - (x-3), & x \geq 3 \\ 5 - (-x+3), & x < 3 \end{cases}$$

$$= \begin{cases} 5 - x + 3, & x \geq 3 \\ 5 + x - 3, & x < 3 \end{cases} = \begin{cases} 8 - x \text{ (5)}, & x \geq 3 \\ x + 2 \text{ (5)}, & x < 3 \end{cases}$$

$$\lim_{x \rightarrow 3^-} x+2 = \boxed{5}$$

$$\& \lim_{x \rightarrow 3^+} 8-x = 5$$

$$\therefore f(3^-) = f(3^+) = 5 \quad \text{existence (1)}$$

$$f(3) = 8 - 3 = 5$$

$$f(3) = 3 + 2 = 5$$

$$\therefore \lim_{x \rightarrow 3} f(x) = f(x) \text{ (2)}$$

$\therefore f(x)$ is cont.

Discuss the continuity of the function $f : f(x) = \begin{cases} 5x-6, & x \geq 3 \\ 3x+4, & x < 3 \end{cases}$ at $x=3$

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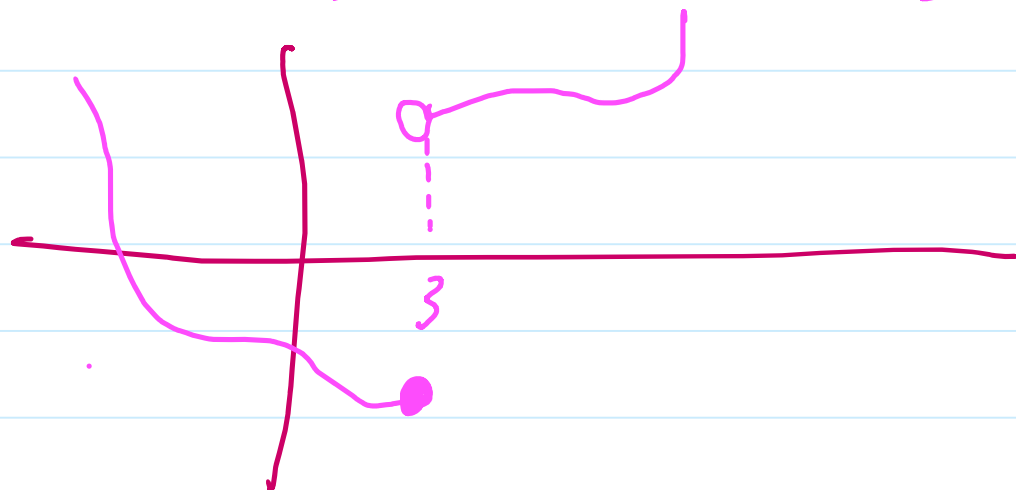
$$\lim_{x \rightarrow 3^-} 3x+4 = 9+4 = \boxed{13}$$

$$\lim_{x \rightarrow 3^+} 5x-6 = 15-6 = \boxed{9}$$

$$f(3^-) \neq f(3^+)$$

$\therefore f(x)$ is not cont. at $x=3$

$$f(a) = \lim_{x \rightarrow a} f(x)$$



Definition

If $f(x)$ is defined on $[a, b]$, and

1- $f(x)$ is continuous on $]a, b[$

2- $\lim_{x \rightarrow a^+} f(x) = f(a)$, $\lim_{x \rightarrow b^-} f(x) = f(b)$

then f is continuous on $[a, b]$

$f(x) = \frac{3}{x-1}$
 $\mathbb{R} - \{1\}$

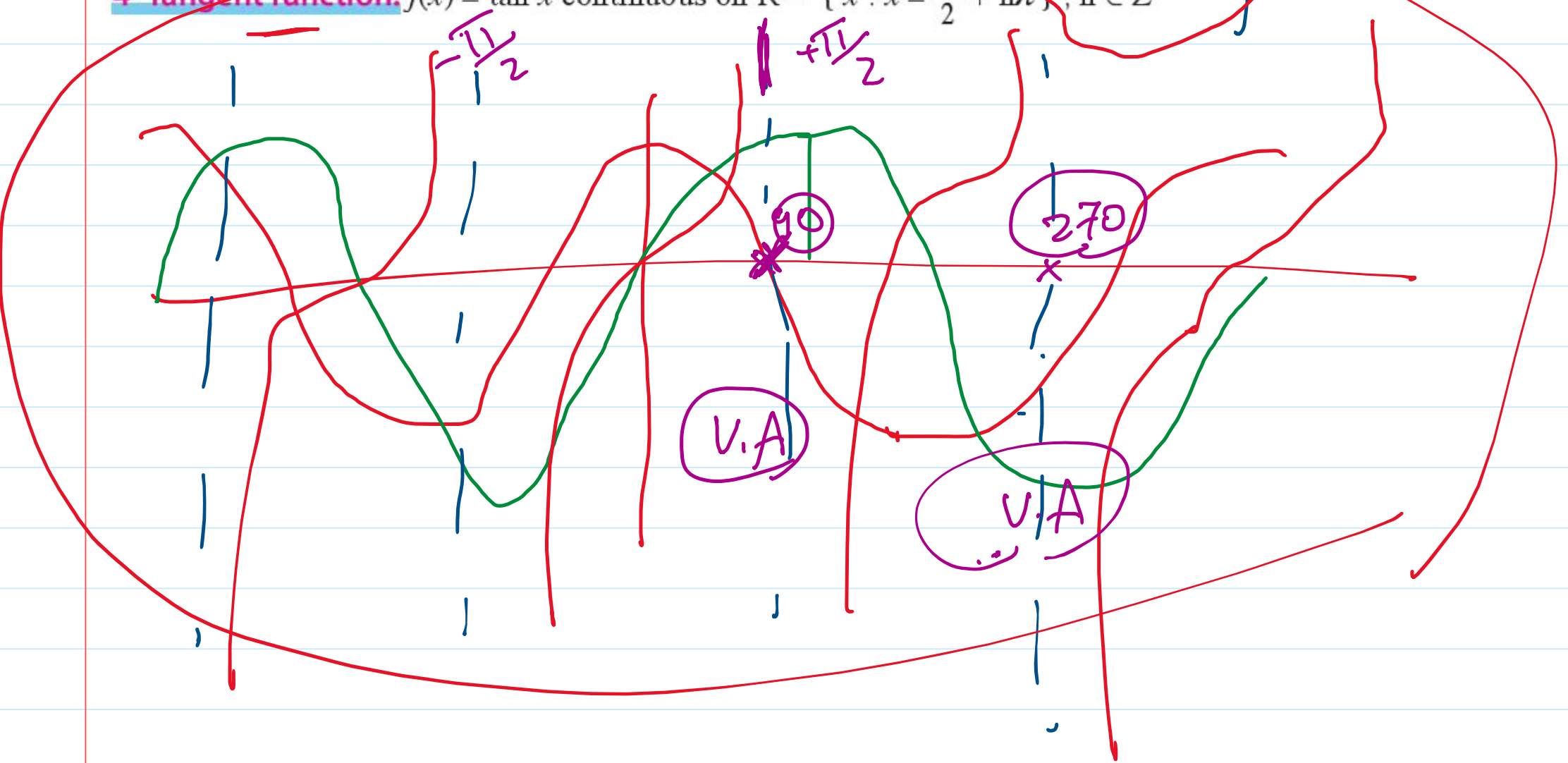
Depending on the above definition and the limits of the functions, we can declare some of the continuous functions

1- **Polynomial function:** continuous on \mathbb{R} or on its domain.

2- **Rational function:** continuous on \mathbb{R} except set of zeros of the denominator.

3- **Sine and cosine function:** continuous on \mathbb{R} .

4- **Tangent function:** $f(x) = \tan x$ continuous on $\mathbb{R} - \{x : x = \frac{\pi}{2} + n\pi\}, n \in \mathbb{Z}$



Continuity_3

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Discuss the continuity of the function $f: f(x) = \frac{2x^2 - 7x - 4}{x - 4}$ at $x = 4$

, if f is discontinuous, can we redefine the function f to become continuous at $x = 4$?

Solution

$$\lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{x - 4} = \lim_{x \rightarrow 4} \frac{(x-4)(2x+1)}{(x-4)}$$

$$= \lim_{x \rightarrow 4} 2x + 1 = 9$$

$f(4) = \text{error}$

$\therefore f(4)$ is undef at 4 $\therefore f(x)$ is not cont.
 $\therefore f(4) \neq \lim_{x \rightarrow 4} f(x) \therefore f$ is not cont.

$$\therefore f(x) = \begin{cases} \frac{2x^2 - 7x - 4}{x - 4}, & x \neq 4 \\ 9, & x = 4 \end{cases}$$

Can we redefine the function f :

$$f(x) = \begin{cases} \frac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

to become continuous at $x = 2$?

$|x-2| = (x-2)(-x+2)$
 $x > 2$ $x < 2$

$$f(x) = \begin{cases} \frac{x-2}{x-2}, & x > 2 \\ \frac{-(x-2)}{x-2}, & x < 2 \\ 0, & x = 2 \end{cases} = \begin{cases} 1, & x > 2 \\ -1, & x < 2 \\ 0, & x = 2 \end{cases}$$

$$= \begin{cases} 1, & x > 2 \\ 0, & x = 2 \\ -1, & x < 2 \end{cases}$$

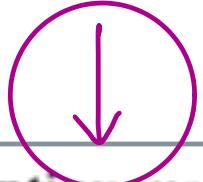
$f(2^+) = 1$
 $f(2^-) = -1$ \neq not exist

we can redefine to become cont. f^n at $x = 2$

Continuity_4

Thursday, November 13, 2025 3:54 PM

If the function $f : f(x) = \begin{cases} \frac{x \cos 5x}{\sin 3x + \tan 4x} & , 0 < x < \frac{\pi}{8} \\ 2b & , x = 0 \\ 3a - \frac{2}{7} \cos x & , x < 0 \end{cases}$



is continuous at $x = 0$ - find the values of a and b

$$\lim_{x \rightarrow 0^+} \frac{x \cos 5x}{\sin 3x + \tan 4x} = \lim_{x \rightarrow 0^+} \frac{\cos 5x}{\frac{\sin 3x}{x} + \frac{\tan 4x}{x}} = \frac{1}{3 + 4} = \frac{1}{7}$$

$$\lim_{x \rightarrow 0^-} 3a - \frac{2}{7} \cos x = 3a - \frac{2}{7}$$

$$3a - \frac{2}{7} = \frac{1}{7}$$

$$3a = \frac{3}{7}$$

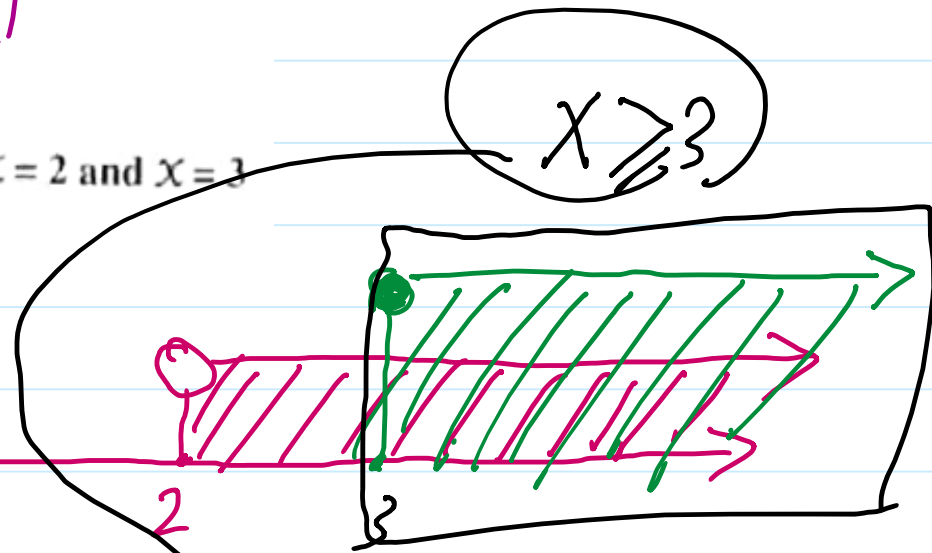
$$a = \frac{1}{7}$$

$$2b = \frac{1}{7}$$

$$b = \frac{1}{14}$$

If $f(x) = \begin{cases} x^2 - 2x + 2 & , x \leq 2 \\ |x - 3| & , x > 2 \end{cases}$ - discuss the continuity of f at $x = 2$ and $x = 3$

Sc $|x - 3| \begin{cases} x - 3 & , x \geq 3 \\ -(x - 3) & , x < 3 \end{cases}$



$$f(x) = \begin{cases} x^2 - 2x + 2 & , x \leq 2 \\ x - 3 & , x \geq 3 \\ -(x - 3) & , 2 < x < 3 \end{cases}$$

