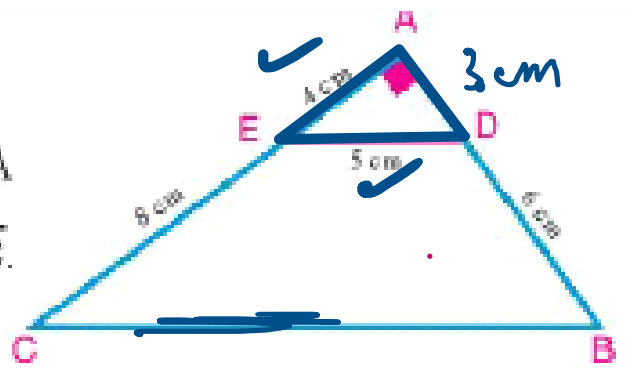


Example

- 3 In the figure opposite: ABC is a right angled triangle at A
 A Prove that: $\overline{DE} \parallel \overline{BC}$.
 B Find the length of \overline{BC} .



Solution

$$\frac{AD}{AB} = \frac{AE}{AC} \quad / \quad \frac{AB}{DB} = \frac{AC}{EC} \quad / \quad \frac{AE}{DB} = \frac{AD}{EC}$$

In $\triangle BAC$: $\therefore m(\angle A) = 90^\circ$ $\therefore BC^2 = AB^2 + AC^2$ ←

in $\triangle DAE$: $\therefore m(\angle A) = 90^\circ$ $\therefore ED^2 = AD^2 + AE^2$

$\therefore 5^2 = 4^2 + AD^2$ $\therefore AD = \sqrt{25 - 16} = \sqrt{9} = 3 \text{ cm}$

$\therefore \frac{AB}{DB} = \frac{9}{6} = \frac{3}{2} \longrightarrow \textcircled{1}$
 $\therefore \frac{AC}{EC} = \frac{12}{8} = \frac{3}{2} \longrightarrow \textcircled{2}$ } from $\textcircled{1}$ & $\textcircled{2}$
 $\therefore \frac{AB}{DB} = \frac{AC}{EC} = \frac{3}{2}$

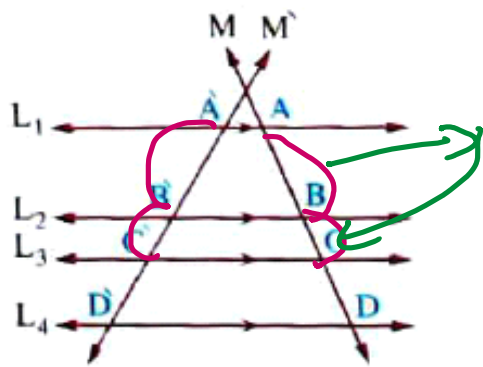
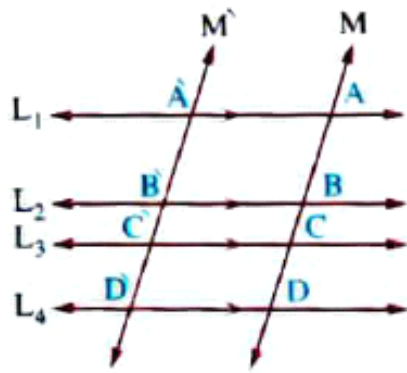
$\therefore \overline{DE} \parallel \overline{BC} \quad \# \textcircled{1}$

$\therefore BC = \sqrt{12^2 + 9^2} = 15 \text{ cm} \quad \# \textcircled{2}$

→ \therefore

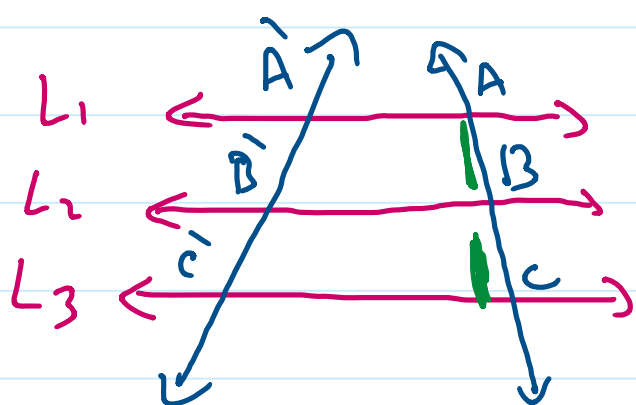
Theorem 2

Given several coplanar parallel lines and two transversals, then the lengths of the corresponding segments on the transversals are proportional.



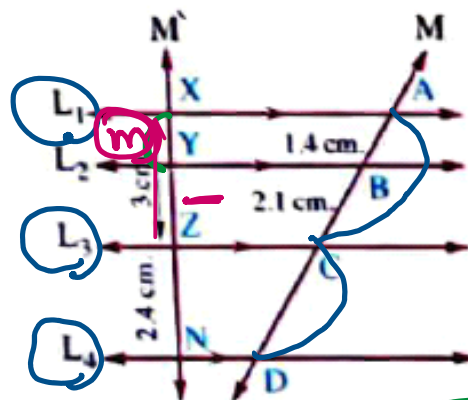
$\therefore L_1 \parallel L_2 \parallel L_3 \parallel L_4$

$\therefore \frac{AB}{BC} = \frac{A'B'}{B'C'}$ (1)



$L_1 \parallel L_2 \parallel L_3 \therefore \frac{AB}{BC} = \frac{A'B'}{B'C'}$

In the opposite figure :
 $L_1 \parallel L_2 \parallel L_3 \parallel L_4$ and
 M, M' are two transversals.
 Use the lengths shown to
 calculate the length of each of \overline{XY} and \overline{CD}



$\therefore L_1 \parallel L_2 \parallel L_3 \parallel L_4$

$\therefore \frac{AB}{BC} = \frac{xy}{yz}$

$\Rightarrow \frac{1.4}{2.1} = \frac{m}{3-m}$

$xz = 3 \text{ cm}$
 $\text{let } xy = m \Rightarrow yz = 3 - m$

$\frac{AC}{DC} = \frac{xz}{zn}$

$\Rightarrow \frac{3.5}{DC} = \frac{3}{2.4}$

$\Rightarrow DC = \frac{2.4 \times 3.5}{3}$

$2.1m = 1.4(3 - m)$

$2.1m = 4.2 - 1.4m$

$\frac{3.5m}{3.5} = \frac{4.2}{3.5} \Rightarrow m = 1.2$

$\therefore xy = 1.2 \text{ cm}$

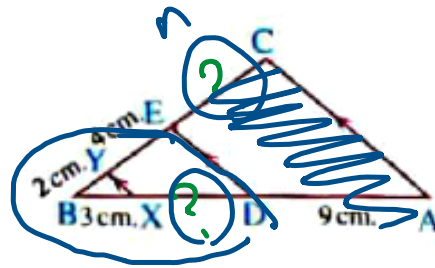
In the opposite figure :

ABC is a triangle ,

$\overline{AC} \parallel \overline{DE} \parallel \overline{XY}$,

AD = 9 cm. , XB = 3 cm. , BY = 2 cm. , EY = 4 cm.

Find : CE and DX



$\therefore \overline{AC} \parallel \overline{DE} \parallel \overline{XY}$

$\therefore \frac{BY}{YC} = \frac{BX}{XA}$

$\frac{BE}{YE} = \frac{BD}{XD}$

$\frac{BC}{BY} = \frac{BA}{BX}$

Let $XD = m$
 $\therefore BD = m + 3$

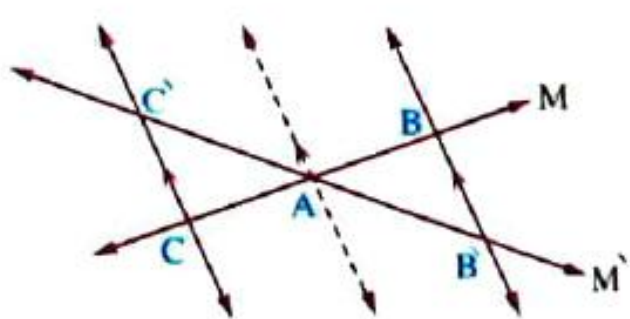
$\frac{BE}{EC} = \frac{BD}{DA}$

~~$\frac{6}{4} = \frac{m+3}{m}$~~

$\therefore 6m = 4m + 12$
 $\therefore m = 6$ $\therefore XD = 6 \text{ cm}$ $\frac{2m}{2} = \frac{12}{2}$

Two special cases

1 If the two lines M and M' intersect at the point A and $\overline{BB'} \parallel \overline{CC'}$



, then $\frac{AB}{AC} = \frac{AB'}{AC'}$
 and conversely if $\frac{AB}{AC} = \frac{AB'}{AC'}$, then $\overline{BB'} \parallel \overline{CC'}$

2 Talies' special theorem :

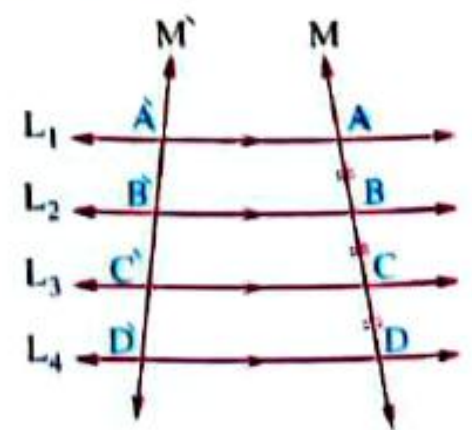
If the lengths of the segments on the transversal are equal , then the lengths of the segments on any other transversal will be also equal.

In the opposite figure :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$,

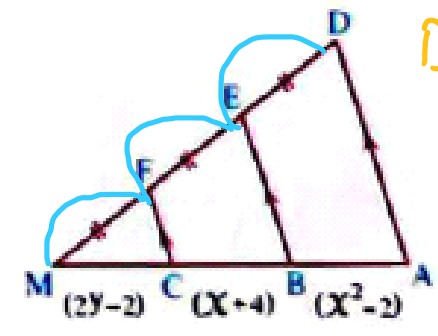
M and M' are two transversals to them

and if $AB = BC = CD$, then $\hat{A}\hat{B} = \hat{B}\hat{C} = \hat{C}\hat{D}$



In the opposite figure :

$\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$, $DE = EF = FM$, find the value of each of x and y
 (lengths are measured in centimetres)



$$CB = -2 + 4 = 2$$

$$BA = (-2)^2 - 2 = 4 - 2 = 2$$

$$\therefore \overline{AD} \parallel \overline{BE} \parallel \overline{CF}$$

$$\therefore MF = FE = ED$$

$$\therefore 2y - 2 = x + 4 = x^2 - 2$$

$$\therefore MC = CB = BA$$

$$x^2 - 2 = x + 4 \Rightarrow x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$AB = 1$$

$$BC = 7$$

$$x = 3$$

$$x = -2$$

refused

$$\therefore 2y(-2) = 2$$

$$\frac{2y}{2} = \frac{4}{2}$$

$$y = 2$$

4 In the figure opposite: $\overline{AB} \parallel \overline{DE}$ and $\overline{AE} \cap \overline{BD} = \{C\}$
 $AC = 6\text{cm}$, $BC = 4\text{cm}$ and $CD = 3\text{cm}$.
 Find the length \overline{AE}

