

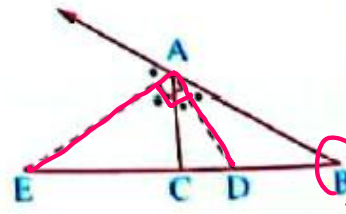
i.e. The interior and exterior bisectors for any angle in the triangle are perpendicular

1 In the opposite figure :

If \overline{AD} , \overline{AE} are the bisectors of the angle A and the exterior angle of ΔABC at A respectively

, then $\frac{BD}{DC} = \frac{AB}{AC}$, $\frac{BE}{EC} = \frac{AB}{AC}$ $\therefore \frac{BD}{DC} = \frac{BE}{EC}$

\therefore The base \overline{BC} is divided internally at D, externally at E by the same ratio (AB : AC) and we notice that : the two bisectors \overline{AD} and \overline{AE} are perpendicular.



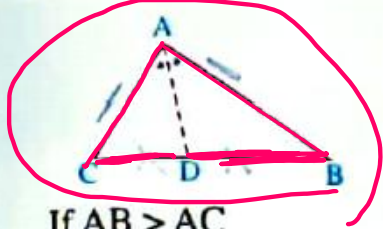
$\frac{BD}{DC} = \frac{BA}{AC}$ \rightarrow int. ①

$\frac{BE}{EC} = \frac{BA}{AC}$ \rightarrow ext. ②

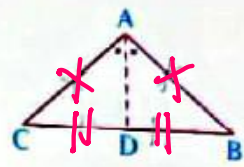
$\therefore \frac{BD}{DC} = \frac{BE}{EC}$ \rightarrow result

i.e. $m(\angle DAE) = 90^\circ$

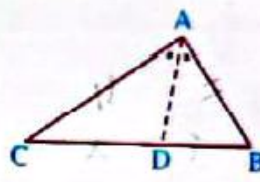
2 If \overline{AD} bisects $\angle BAC$ and intersects \overline{BC} at D, then D takes one of the following :



If $AB > AC$
 , then $BD > DC$
i.e. D is nearer to C than to B



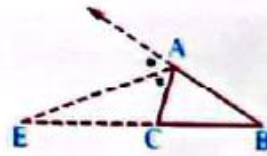
If $AB = AC$
 , then $BD = DC$
i.e. D is equidistant from each of B and C



If $AB < AC$
 , then $BD < DC$
i.e. D is nearer to B than to C

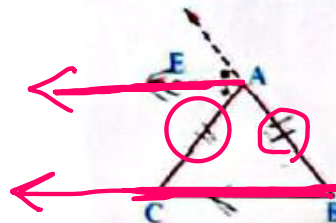
3 If \overline{AE} bisects the exterior angle of ΔABC at A, where $E \notin \overline{BC}$, then E takes one of the following cases :

① If $AB > AC$, then $BE > EC$ **i.e.** $E \in \overline{BC}$

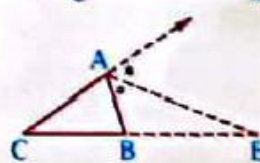


② If $AB = AC$, then $\overline{AE} \parallel \overline{BC}$

i.e. the exterior bisector of the vertex of isosceles triangle is paralleling to the base.



③ If $AB < AC$, then $BE < EC$ **i.e.** $E \in \overline{CB}$



Angle bisector and proportional parts_4

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ABC is a triangle in which $AB = 8 \text{ cm}$, $AC = 6 \text{ cm}$, $BC = 7 \text{ cm}$, draw \overline{AD} to bisect $\angle A$ and intersect \overline{BC} at D, draw \overline{AE} to bisect the exterior angle A and intersect \overline{BC} at E

Find the length of: \overline{DE}

$\therefore \overline{AD}$ bisect $\angle BAC$

$$\therefore \frac{BD}{DC} = \frac{BA}{AC}$$

Let $DC = x \quad \therefore BD = 7 - x$

$$\frac{7-x}{x} = \frac{8}{6} \quad 8x = 42 - 6x$$

$$\frac{14x}{14} = \frac{42}{14} \quad x = 3 \quad \boxed{DC = 3 \text{ cm}}$$

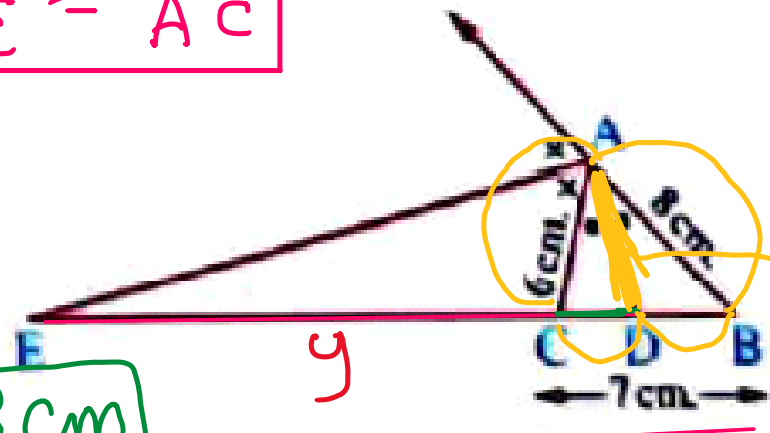
$\therefore \overline{EA}$ bisect ^{external} angle of A

$$\therefore \frac{BE}{EC} = \frac{BA}{CA}$$

Let $EC = y$ $\frac{y+7}{y} \neq \frac{8}{6}$

$$\therefore 8y - 6y = 6y + 42 \quad \therefore \frac{2y}{2} = \frac{42}{2} \quad y = 21$$

$$\therefore EC = 21 \text{ cm} \quad \therefore ED = 21 + 3 = 24 \text{ cm} \quad \#$$



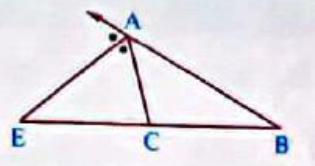
If \overline{AD} bisects $\angle A$ in ΔABC internally and intersects \overline{BC} at D

, then $AD = \sqrt{AB \times AC - BD \times DC}$ #

In the opposite figure :

If \overline{AE} bisects $\angle BAC$ externally and intersects \overline{BC} at E

, then $AE = \sqrt{BE \times EC - AB \times AC}$

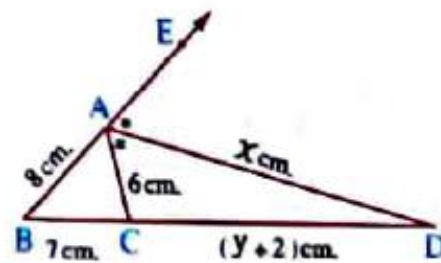


In the opposite figure :

ABC is a triangle in which $AB = 8 \text{ cm}$,

$BC = 7 \text{ cm}$, $AC = 6 \text{ cm}$, \overline{AD} bisects $\angle A$ externally.

Find the value of each of: x, y



Converse of theorem 3

In the opposite two figures :

• If $D \in \overline{BC}$ (Fig. 1)

such that : $\frac{BD}{DC} = \frac{BA}{AC}$

, then \overline{AD} bisects $\angle BAC$

• If $D \in \overline{BC}$, $D \notin \overline{BC}$ (Fig. 2)

such that : $\frac{BD}{DC} = \frac{BA}{AC}$

, then \overline{AD} bisects the exterior angle of ΔABC at A

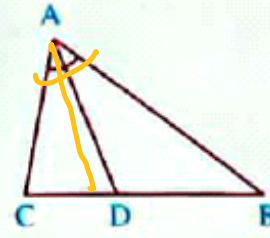


Fig. (1)

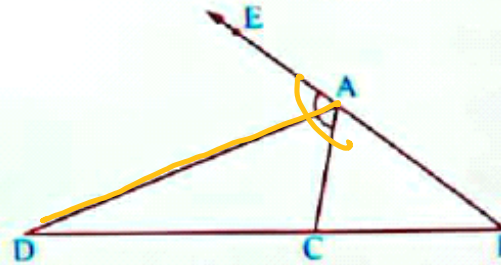


Fig. (2)

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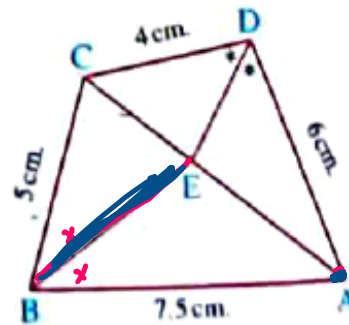
In the opposite figure :

ABCD is a quadrilateral in which $AB = 7.5$ cm.

, $BC = 5$ cm. , $CD = 4$ cm. , $AD = 6$ cm.

, \overline{DE} bisects $\angle ADC$ and intersects \overline{AC} at E

Prove that : \overline{BE} bisects $\angle ABC$



$$\frac{AE}{EC} = \frac{AB}{BC}$$

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$\therefore DE$ bisects $\angle CDA$

$$\therefore \frac{AE}{EC} = \frac{AD}{DC} = \frac{6}{4} = \frac{3}{2}$$

$$\therefore \boxed{\frac{AE}{EC} = \frac{3}{2}} \rightarrow \textcircled{1}$$

$$\therefore \boxed{\frac{AB}{BC} = \frac{3}{2}} \rightarrow \textcircled{2}$$

$$\therefore \frac{AB}{BC} = \frac{7.5}{5} = \frac{3}{2}$$

from $\textcircled{1}$ & $\textcircled{2}$

$$\therefore \frac{AE}{EC} = \frac{AB}{BC} = \frac{3}{2}$$

$\therefore \overline{BE}$ bisects $\angle ABC$ #

Angle bisector and proportional parts_6

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ABC is an isosceles triangle in which $AB = AC$, $D \in \overline{BC}$, where $BC = CD$, draw the bisector of the angle ABC to intersect \overline{AC} at E, draw $\overline{EF} \parallel \overline{BC}$ and intersects \overline{AD} at F
 Prove that : \overline{CF} bisects $\angle ACD$

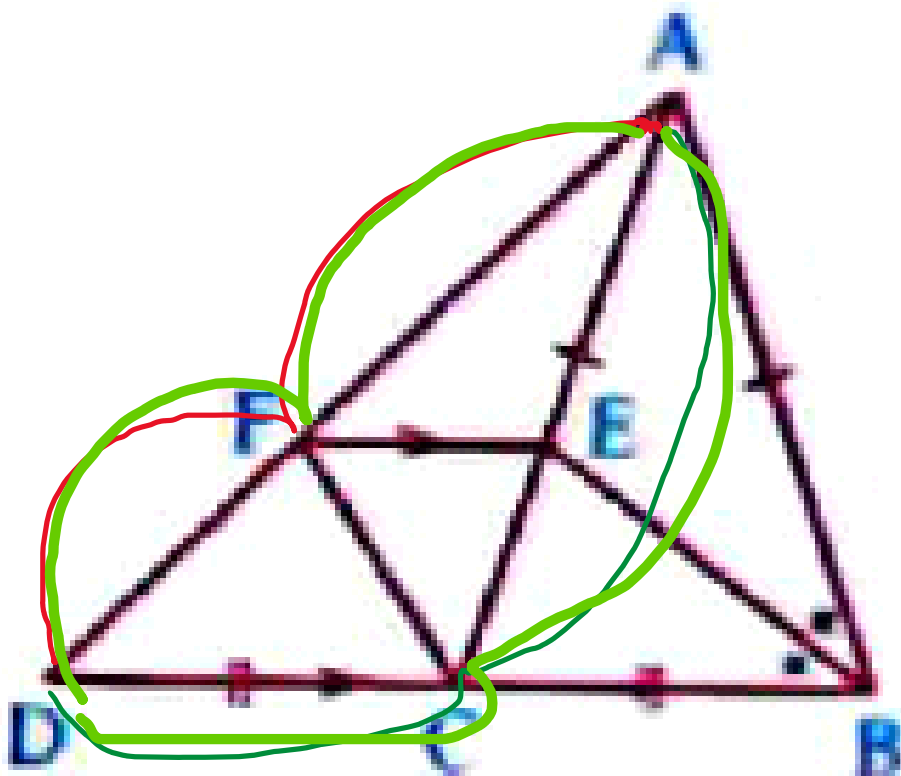
$\therefore \overline{BE}$ bisect $\angle ABC$

$$\therefore \frac{CE}{EA} = \frac{CB}{BA} \rightarrow (1)$$

$$\therefore AB = AC \quad \&$$

$$BC = CD$$

$$\therefore \frac{CE}{EA} = \frac{CD}{AC} \leftarrow$$



$$\therefore \overline{EF} \parallel \overline{BC} \rightarrow \frac{AE}{EC} = \frac{AF}{FD} \rightarrow (2) \text{ R.H.S}$$

L.H.S

$$\rightarrow \frac{AE}{EC} = \frac{AC}{CD} \rightarrow (3)$$

from (2) & (3)

$$\therefore \frac{AF}{FD} = \frac{AC}{CD}$$

$\therefore \overline{CF}$ bisect $\angle ACD$

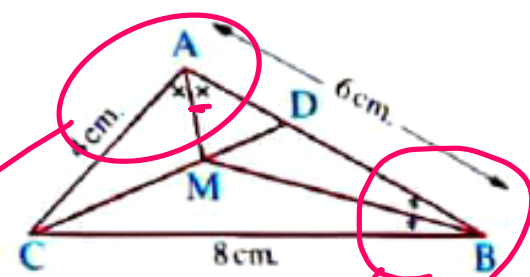
Angle bisector and proportional parts_8

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In the opposite figure :

ABC is a triangle in which $AB = 6 \text{ cm}$, $AC = 4 \text{ cm}$,
 $BC = 8 \text{ cm}$, \overline{BM} bisects $\angle ABC$, \overline{AM} bisects $\angle BAC$

Find the length of : \overline{AD}



L.H.S

$$\frac{DM}{MC} = \frac{DA}{AC} \quad \text{--- (1)}$$
$$\frac{DM}{MC} = \frac{DB}{BC} \quad \text{--- (2)}$$

Let $AD = x$
 $BD = 6 - x$

$$\therefore \frac{DA}{AC} = \frac{DB}{BC}$$

$$\frac{x}{4} = \frac{6-x}{8}$$

$$\Rightarrow 8x = 24 - 4x$$
$$\frac{12x}{12} = \frac{24}{12} \quad \boxed{x=2}$$

$$AD = 2 \text{ cm}$$

$$BD = 6 - 2 = 4 \text{ cm}$$