

Rational Exponent_1

Thursday, November 27, 2025 8:12 AM

$$\begin{aligned} (-2)^6 &= 2^6 \\ (-2)^5 &= -2^5 \end{aligned}$$

Find the solution set in \mathbb{R} for each of the following :

(1) $3x^5 = -96$

(2) $x^6 = -64 = -(2^6)$

(3) $(x-2)^4 = 81$

(4) $x^{\frac{3}{4}} = 27$

(5) $\sqrt[5]{x^2} = 1$

(6) $\sqrt[4]{(3x+2)^3} = 8$

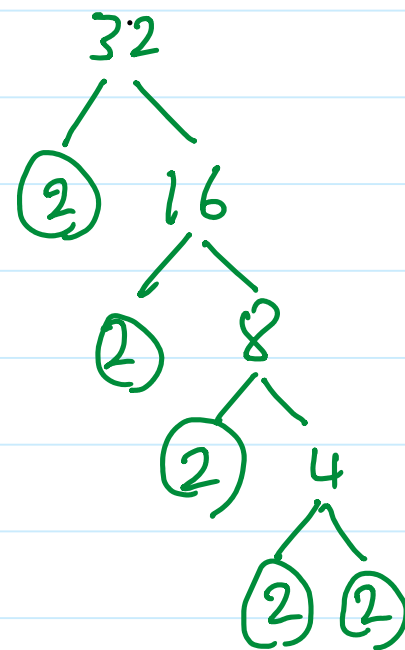
(7) $x^{\frac{4}{3}} - 5x^{\frac{2}{3}} + 4 = 0$

(8) $(x^2 - 4x - 13)^{\frac{3}{5}} = 8$

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$$\frac{3x^5}{3} = \frac{-96}{3}$$

$$\begin{aligned} x^5 &= -32 = (-2)^5 \\ x^5 &= (-2)^5 \end{aligned}$$



(2) $x^6 = -2^6$
 ~~$x = -2$~~
 S.S. = \emptyset
 no sol.

$\therefore x = -2$

$\therefore \text{S.S.} = \{-2\}$

(4) $x^{\frac{3}{4}} = 27$

$$\left(\sqrt[4]{x}\right)^3 = \sqrt[4]{x^3}$$

$$\left(\sqrt[4]{x^3}\right)^4 = (27)^4$$

$$3\sqrt{x^3} = \sqrt[3]{531441}$$

$$x = 81$$

$$\sqrt[3]{\left(\sqrt[4]{x}\right)^3} = \sqrt[3]{27}$$

$$\left(\sqrt[4]{x}\right)^4 = 3^4$$

$$x = 81$$

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(6) $\sqrt[4]{(3x + 2)^3} = 8$

$x^{\frac{4}{3}} - 5x^{\frac{2}{3}} + 4 = 0$

Let $x^{\frac{1}{3}} = y$

$y^4 - 5y^2 + 4 = 0$

$(y^2 - 4)(y^2 - 1) = 0$

$y^2 = 4$ $y^2 = 1$

$y = \pm 2$

$y = \pm 1$

$\sqrt[3]{x^{\frac{1}{3}}} = \sqrt[3]{\pm 2}$

$\sqrt[3]{x^{\frac{1}{3}}} = \sqrt[3]{\pm 1}$

$x = \sqrt[3]{2}$

$x = \sqrt[3]{1} = 1$

$= \sqrt[3]{-2}$

$x = \sqrt[3]{-1} = -1$

S.S = $\{-1, \sqrt[3]{-2}, \sqrt[3]{2}, 1\}$

Laws of exponents

• For every $m, n \in \mathbb{Z}$ and $a, b \in \mathbb{R} - \{-1, 0, 1\}$ we have:

(1) If $a^n = 1$, then $n = \text{zero}$

(2) If $a^m = a^n$, then $m = n$

(3) If $a^n = b^n$,

- if n is an odd number, then $a = b$
- if n is an even number, then $a = \pm b$
- if $a \neq b$, then $n = \text{zero}$

Find the value of x that satisfies each of the following equations:

(1) $2^{x+5} = 8$

(2) $3^{x^2-4} = 1$

(3) $4^{x+2} = x^{x+2}$

(4) $4^{x-3} = 3^{2x-6}$

(5) $\left(\frac{2}{3}\right)^{|x-5|} = \left(3\frac{3}{8}\right)^{-2}$

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$2^{x+5} = 8 \cdot 2^3$

$2^{x+5} = 2^3$

$x + 5 = 3$

$x = -2$

(4) $4^{x-3} = 3^{2x-6}$

$4^{x-3} = 3^{2x-6}$

$x - 3 = 0$

$x = 3$

(5) $\left(\frac{2}{3}\right)^{|x-5|} = \left(3\frac{3}{8}\right)^{-2}$

$\left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{8}{27}\right)^2$

$\left(\frac{2}{3}\right)^{|x-5|} = \left(\frac{2}{3}\right)^6$

$|x-5| = 6$

$x - 5 = 6 \Rightarrow x = 11$
 $x - 5 = -6 \Rightarrow x = -1$

$\left(\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right)^6$

Find in \mathbb{R} the S.S. of each of the following equations :

$$(1) 2^x \times \sqrt[3]{4} = (\sqrt[3]{16})^{-1}$$

$$(2) 4^{x^2-1} = 8^{-x}$$

$$(3) \frac{3^{3x} + 3^{2x} + 3^x}{3^{2x} + 3^x + 1} = \frac{1}{9}$$

$$\textcircled{1} \quad 2^x \cdot 2^{2/3} = (2^{4/3})^{-1}$$

$$\frac{x+2/3}{3} = \frac{-4/3}{3}$$

$$\textcircled{3} \quad \frac{(3^x)^3 + (3^x)^2 + 3^x}{(3^x)^2 + 3^x + 1} = \frac{1}{9}$$

$$\text{Let } 3^x = y$$

$$\frac{y^3 + y^2 + y}{y^2 + y + 1} = \frac{1}{9}$$

$$\frac{y(y^2 + y + 1)}{y^2 + y + 1} = \frac{1}{9}$$

$$y = \frac{1}{9}$$

$$3^x = \frac{1}{9}$$

$$3^x = 3^{-2}$$

$$\boxed{x = -2}$$

$$y^2 + 125 = 30y$$

Find in \mathbb{R} the S.S. of each of the following equations :

(1) $2^{x+1} + 2^{x-1} = 5$

(2) $5^x + \frac{125}{5^x} = 30$ $\neq y$

$$2^{x-1} (2^2 + 1) = 5$$

$$2^{x-1+2} = 2^{x+1}$$

$$2^{x-1} \cdot 2^2$$

$$2^{x-1} \cdot 4 = 5$$

$$2^{x-1} = \frac{5}{4}$$

$$x-1 = 0 \quad \boxed{x=1}$$