

Exponential Function and its application_1

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$a \in \mathbb{R}^+ - \{1\}$

then the function $f: \mathbb{R} \rightarrow \mathbb{R}^+$ where $f(x) = a^x$ is called an exponential function whose base is 'a'

For example :

- $f: f(x) = 3^x$ is an exponential function whose base = 3 and its power = x
- $f: f(x) = (\frac{1}{2})^{x+1}$ is an exponential function whose base = $\frac{1}{2}$ and its power = $x+1$

$f(x) = x^2 + 5$

$= 3x^3$

$f(x) = 2^x \rightarrow \text{exp. f.}$

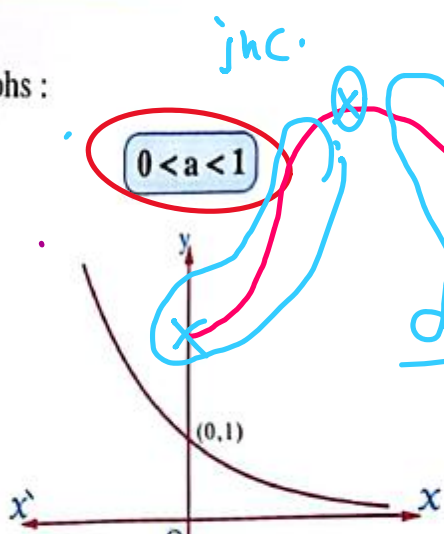
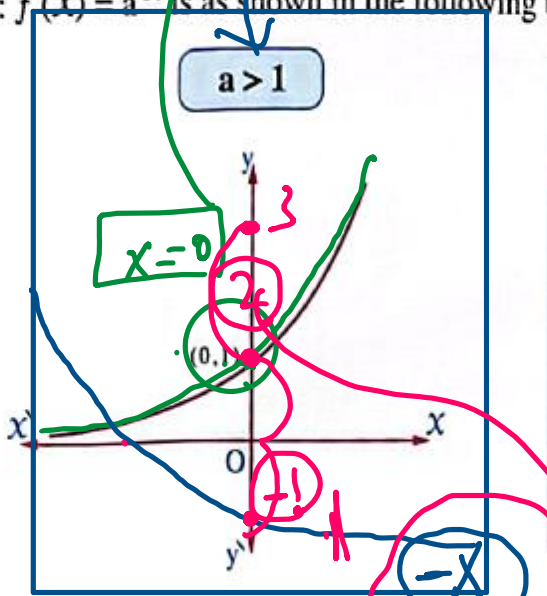
$f(x) = \begin{matrix} 2^x \\ 5^x \\ 4^x \\ 20^x \end{matrix}$

$f(x) = \begin{matrix} (\frac{1}{3})^x \\ (\frac{1}{2})^x \\ (\frac{3}{4})^x \end{matrix}$

\mathbb{R}

The graphical representation of the exponential function

The general diagram of the graph of the function $f: f(x) = a^x$ is as shown in the following two graphs :



Domain = \mathbb{R}
 Range = $]0, \infty[$
 - one to one fun.
 increase.

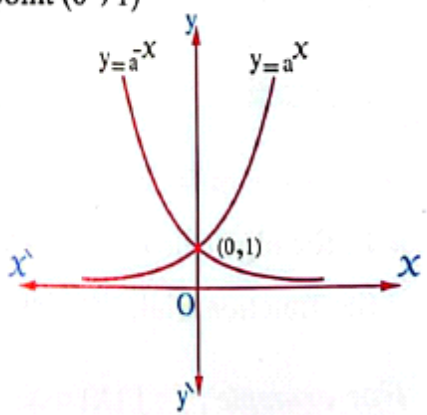
$f(x) = a^{-x}$

$f(x) = (\frac{1}{2})^{-x}$

$f(x) = a^{-x} - 2$

Some properties of the exponential function $f: f(x) = a^x$

- The domain = \mathbb{R}
- The range = \mathbb{R}^+ and its curve lies completely above the X-axis.
- The function is increasing on its domain \mathbb{R} when $a > 1$ and is called an exponential growth function, its coefficient is a and the curve of the function approach to X-axis by the decreasing of the value of X
- The function is decreasing on its domain \mathbb{R} when $0 < a < 1$ and is called an exponential decay function, its coefficient is a and the curve of the function approach to X-axis by the increasing of the value of X
- The curve of the exponential function passes through the point (0, 1)
- $f: f(x) = a^x$ is one-to-one.
- If $f(x) = a^x$, then $f(-x) = a^{-x} = (\frac{1}{a})^x$ and the curve $y = (\frac{1}{a})^x$ is the image of the curve $y = a^x$ by reflection in y-axis.
- $a^x \rightarrow \infty$ when $x \rightarrow \infty$ where $a > 1$
- $a^x \rightarrow 0$ when $x \rightarrow \infty$ where $0 < a < 1$



Exponential Function and its application_2

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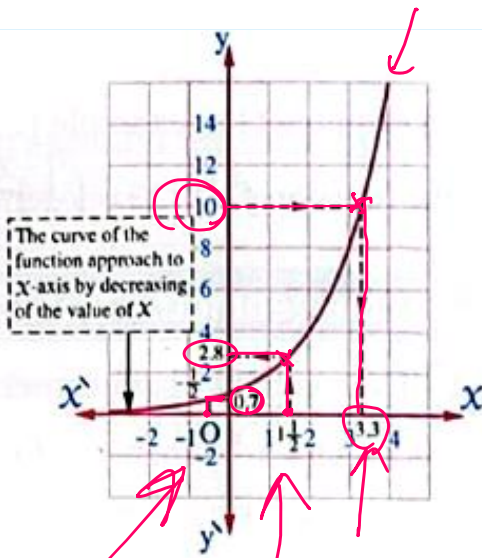
Graph the function $f: \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = 2^x$ taking $x \in [-3, 4]$ and from the graph find an approximated value for each of the following:

(1) $f(1.5), f\left(\frac{-1}{2}\right)$

(2) The value of x when $f(x) = 10$ $y = 10$

$x = 1.5$

x	-3	-2	-1	0	1	2	3	4
$y = 2^x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



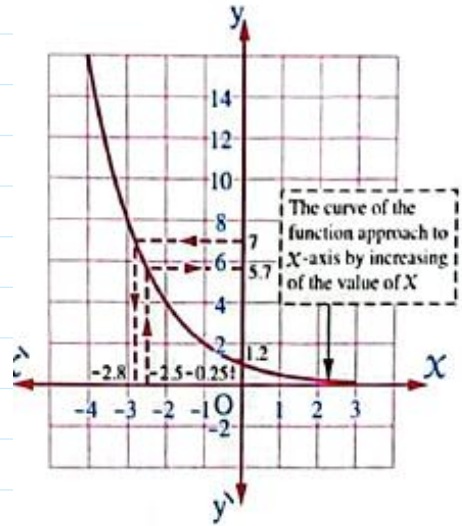
Graph the function $f: \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = \left(\frac{1}{2}\right)^x$ taking $x \in [-4, 3]$, from the graph find an approximated value for each of the following:

(1) $f(-2.5)$

(2) $\sqrt[4]{2}$

(3) The value of x when $\left(\frac{1}{2}\right)^x = 7$

x	-4	-3	-2	-1	0	1	2	3
$y = \left(\frac{1}{2}\right)^x$	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



Some geometric transformations of the function $f(x) = a^x$

1 The vertical displacement to the curve of the exponential

If $f(x) = a^x$, then $y = f(x) + c$

i.e. $y = a^x + c$ is represented graphically by the curve $y = a^x$ by vertical displacement of magnitude $|c|$

* In direction of \vec{Oy} if $c > 0$ (displacement upwards)

* In direction of \vec{Oy} if $c < 0$ (displacement downwards)

2 The horizontal displacement to the curve of the exponential function

If $f(x) = a^x$, then $y = f(x + b)$

i.e. $y = a^{x+b}$ is represented graphically by the curve $y = a^x$ by horizontal displacement of magnitude $|b|$

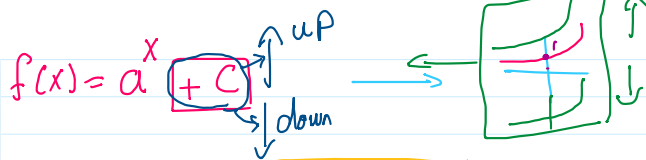
* In direction of \vec{Ox} if $b < 0$

* In direction of \vec{Ox} if $b > 0$

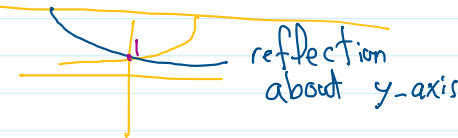
3 The reflection of the curve of the exponential function in X-axis

If $f(x) = a^x$, then the curve of the function $y = -f(x)$

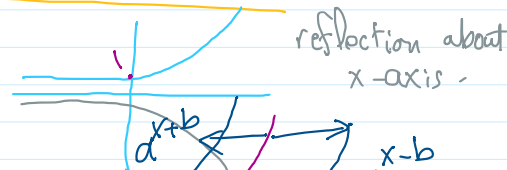
i.e. $y = -a^x$ is the image of the curve $y = a^x$ by reflection in X-axis.



$f(x) = a^{-x}$ ref. about y-axis



$f(x) = a^{x-b}$ ref. about x-axis



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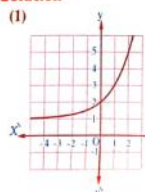
Graph each of the following functions defined by the given rules, from the graph find the domain, the range and determine which of them is increasing and which is decreasing:

(1) $y = 2^x + 1$

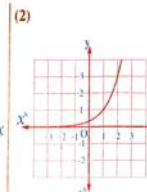
(2) $y = 3^{x-1}$

(3) $y = -(\frac{1}{2})^x$

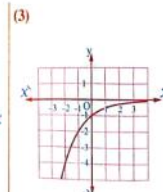
Solution



The domain = \mathbb{R} , the range = $]1, \infty[$, the function is increasing on its domain.



The domain = \mathbb{R} , the range = $]0, \infty[$, the function is increasing on its domain.



The domain = \mathbb{R} , the range = $]-\infty, 0[$, the function is increasing on its domain.

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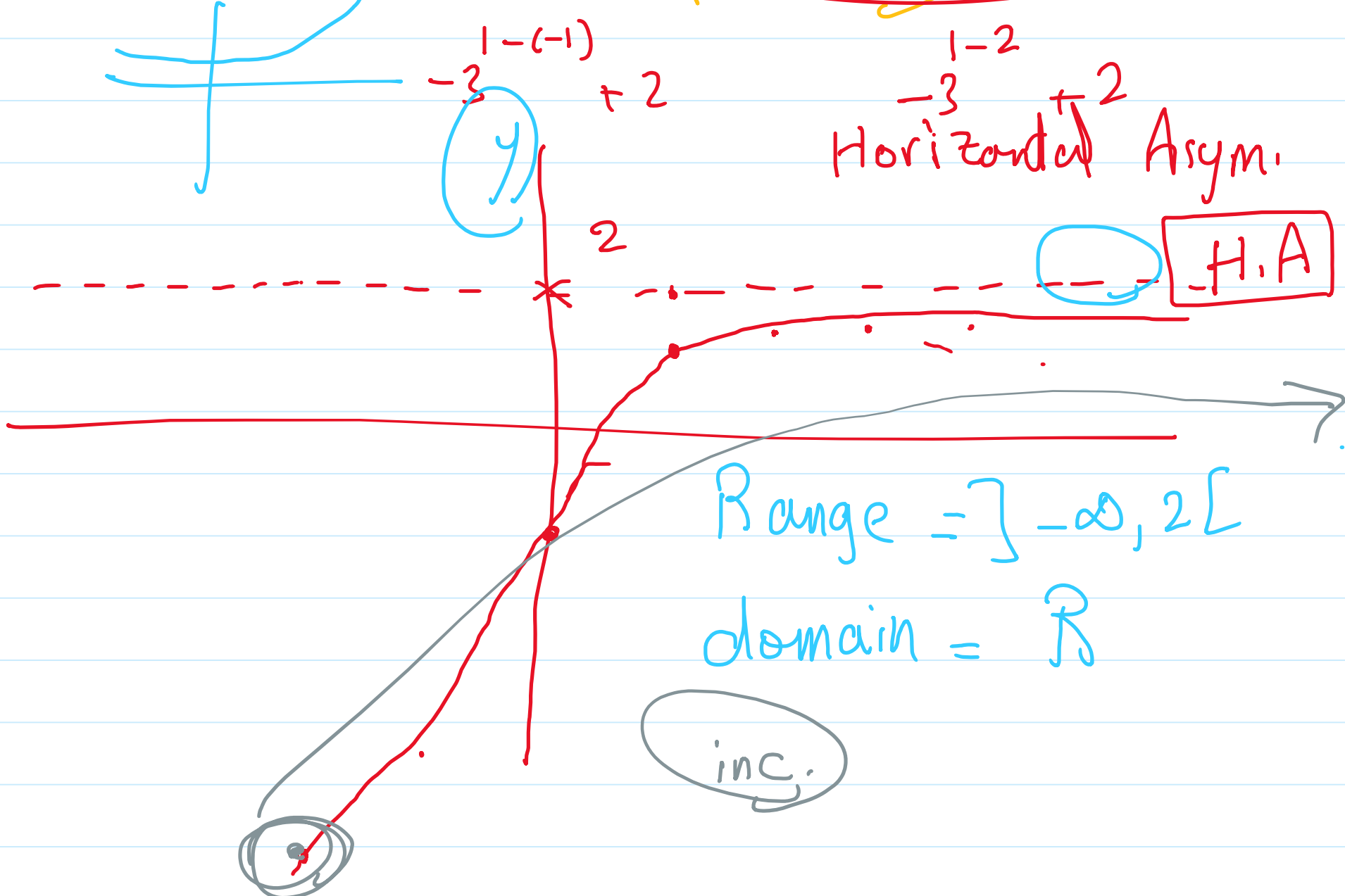
Handwritten notes and graphs illustrating transformations:

- $f(x) = a^{x-b}$ Horizontal displacement: $+b$ left, $-b$ right.
- $f(x) = a^{-x}$ reflection about y-axis.
- $f(x) = -a^x$ reflection about x-axis.
- Graphs showing a^{x+b} and a^{x-b} with arrows indicating horizontal shifts.
- Graph of $f(x) = (\frac{1}{2})^{x-3} + 2$ with a box around $(\frac{1}{2})^{x-3}$ and $+2$.
- Graph showing a curve with a horizontal asymptote at $y=3$ and a point at $(3, 1)$.
- A grid with x and y axes is drawn.

Graph the function defined by the given rule, from the graph find the domain, the range and show whether it is increasing or decreasing: $y = -3^{1-x} + 2$

$-3^{1-x} + 2$

x	-3	-2	-1	0	1	2	3
y	...	-25	-7	-1	1	$1\frac{2}{3}$	$1\frac{8}{9}$



$f(x)$

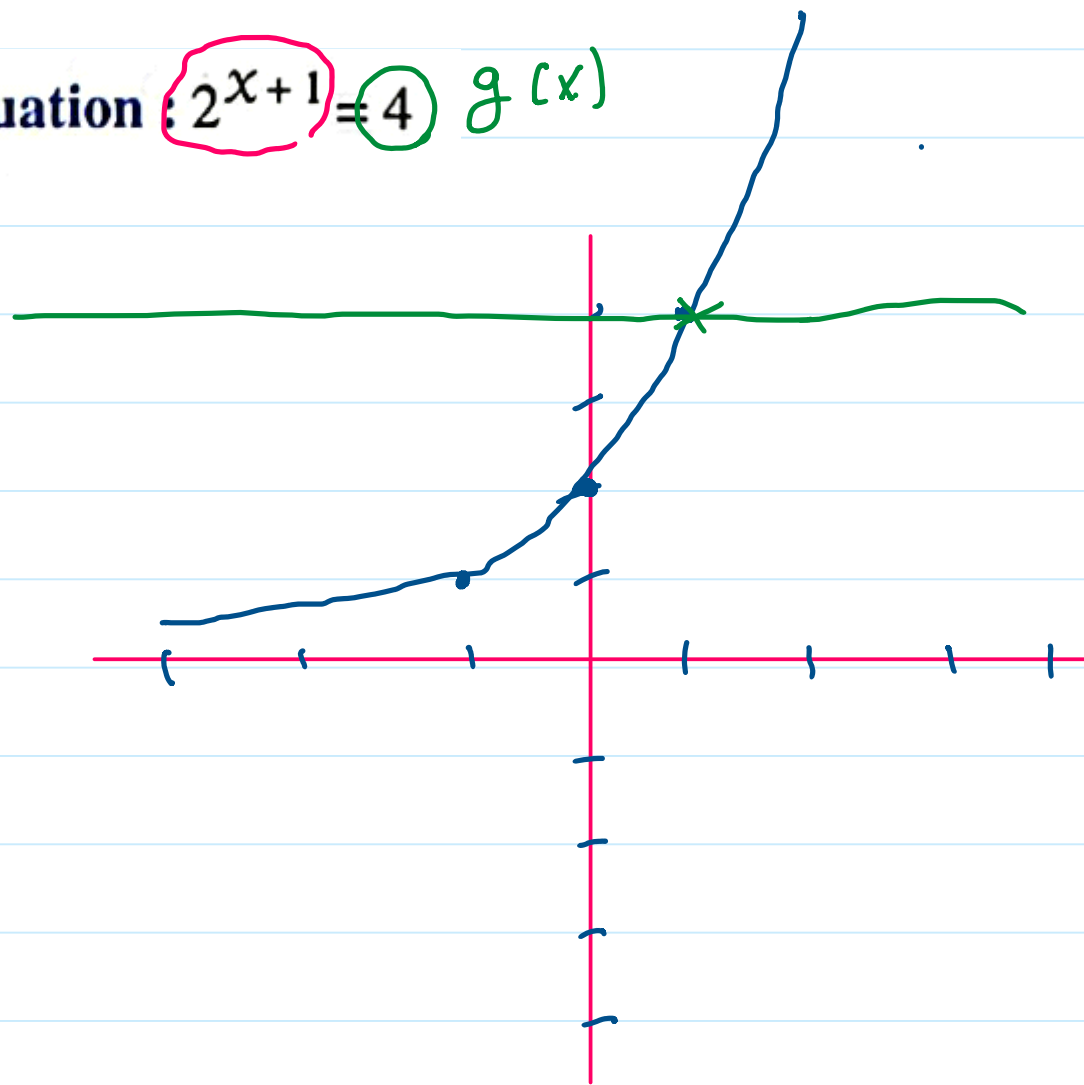
Find graphically in \mathbb{R} the S.S. of the equation: $2^{x+1} = 4$ $g(x)$

$$f(x) = 2^{x+1}$$

x	-3	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

$g(x) = 4$
point of intersection (1, 4)

S.S. = {1}



Exponential Function and its application_6

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If $f(x) = 5^x$, find the value of x if: $f(2x-1) + f(2x+1) = \frac{26}{25}$

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$$= f(2x-1) + f(2x+1) = \frac{26}{25}$$

$$\therefore f(x) = 5^x$$

$$\therefore 5^{2x-1} + 5^{2x+1} = \frac{26}{25}$$

$$5^{2x-1} (1 + 5^2) = \frac{26}{25}$$

$$\cancel{26} \times 5^{2x-1} = \frac{\cancel{26}}{25}$$

$$5^{2x-1} = 25^{-1}$$

$$5^{2x-1} = 5^{-2}$$

$$\therefore 2x - 1 = -2$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$\boxed{x = -\frac{1}{2}}$$

Life applications on the exponential growth and decay

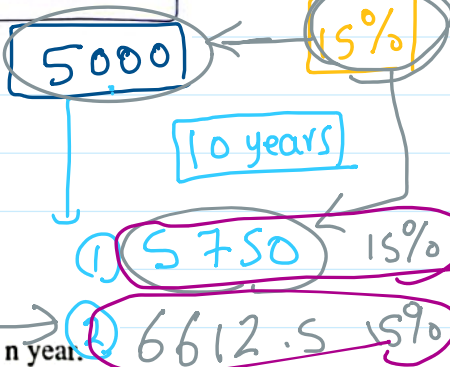
1 Exponential growth

- The function $f : f(t) = a(1+r)^t$ represents the exponential growth with a constant percentage during equal intervals of time, where a is the initial value, r is the percentage of the growth in a constant interval of time, t is the time interval.
- We can deduce this function by studying a phenomena such as the population :
 If the number of population in a city in one of the years is " a " and this number increases annually by constant percentage rate " r ", then the number of population after one year = $a + ra = a(1+r)$, after 2 years = $a(1+r) + ra(1+r) = a(1+r)^2$ and so on, then the number of population after n years = $a(1+r)^n$

$P = a(1+r)^n$

a initial value

0.15 percentage



Wael bought a house by 1350000 L.E. and its price increases at the rate of 2.5% per year :

- Write the exponential function which represents the price of the house after n year.
- Estimate to the nearest pound the price of the house after 6 years.

$P = a(1+r)^n$

$P = 135000(1+0.025)^n$

$P = 135000(1.025)^6 = 1565586 \text{ L.E.}$

Compound interest

bank interest rate

no. of years

$A = P(1 + \frac{r}{n})^{nt}$

initial

times per year

12

15% → year

15% → 6 months

15% → 3 months → $n = 4$

15% → 2 months → $n = 6$

The compound interest



If principal (P) is deposited in one of the banks at interest rate (r) as a percentage and compounded (n) times per year for a period of (t) years, then the accumulated value A is given by : $A = P \left(1 + \frac{r}{n}\right)^{nt}$

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A man deposited a capital of 15000 L.E. in one of the banks with annual compound interest 7%, find the sum of the capital after 10 years in each of the following :

- (1) The interest compounded annually. $\rightarrow n = 1$
- (2) The interest compounded quarter annually. $\rightarrow n = 4$
- (3) The interest compounded monthly. $\rightarrow n = 12$

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$P = 15000$ $r = 7\%$

$t = 10$

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$$A = 15000 \left(1 + \frac{0.07}{1}\right)^{1 \times 10} = \boxed{}$$

$$A = 15000 \left(1 + \frac{0.07}{4}\right)^{4 \times 10} = \boxed{}$$

$$A = 15000 \left(1 + \frac{0.07}{12}\right)^{12 \times 10} = \boxed{}$$

Exponential Function and its application_9

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2 Exponential decay

The function $f : f(t) = a(1-r)^t$ represents the exponential decay where a is the initial value, r is the percentage of the decay in a constant interval of time, t is the time interval.

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pose
decay

0.15

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

The number of infection persons by Hepatitis C is decreased at the rate 15% annually as a result of discovering the new treatment, if the number of infection persons in one of the countries 8000000 infections, write the exponential function which represents the number of infections persons after n years, then estimate the number of the infections persons after 8 years. t

$$f(t) = 8\,000\,000(1 - 0.15)^t$$

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