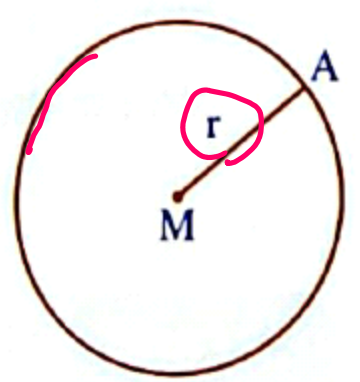


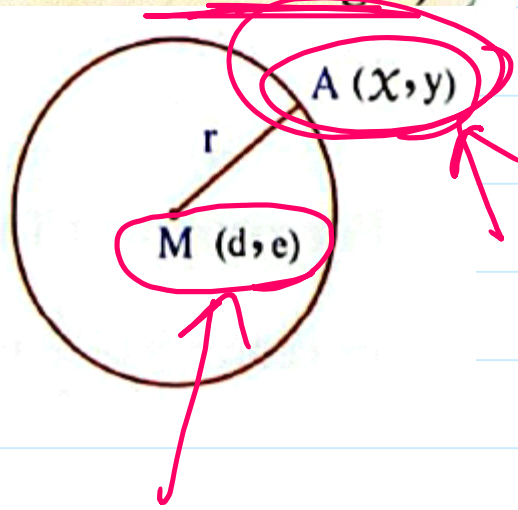
Definition of the circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.



First Equation of the circle (In terms of its centre coordinates and radius length)

$$\sqrt{(x-d)^2 + (y-e)^2} = r$$



①

i.e. $(x-d)^2 + (y-e)^2 = r^2$

L.H.S = R.H.S

The position of the point (x_1, y_1) in respect to the circle

$$C : (x-d)^2 + (y-e)^2 = r^2$$

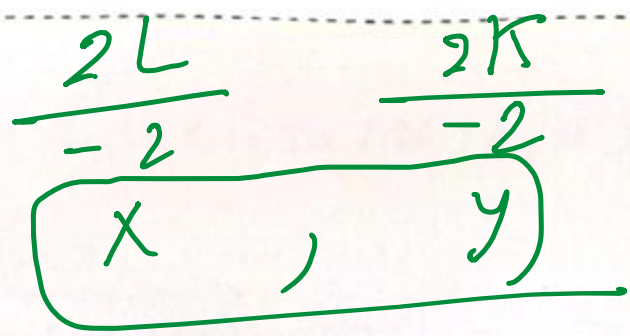
- * If $(x_1 - d)^2 + (y_1 - e)^2 = r^2$, then the point lies on the circle.
- * If $(x_1 - d)^2 + (y_1 - e)^2 > r^2$, then the point lies outside the circle.
- * If $(x_1 - d)^2 + (y_1 - e)^2 < r^2$, then the point lies inside the circle.

Second The general form of the circle equation

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$

The general form of the circle equation is :

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$



Where the centre (M) = $(-L, -K) = (-\frac{1}{2} \text{ coefficient of } x, -\frac{1}{2} \text{ coefficient of } y)$

$$r = \sqrt{L^2 + K^2 - C}, L^2 + K^2 - C > 0$$

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For example : The circle whose equation is : $x^2 + y^2 + 8x - 4y - 16 = 0$

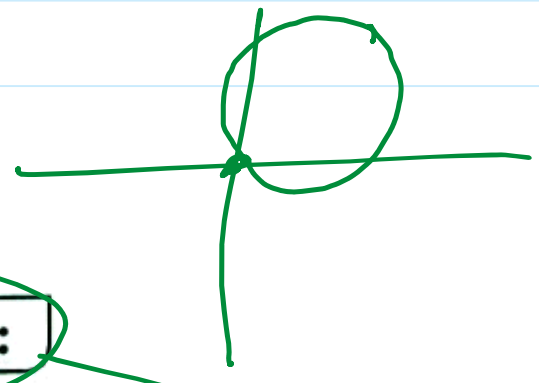
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Special cases

1 Equation of the circle passing through the origin point :

$$x^2 + y^2 + 2Lx + 2Ky = 0$$

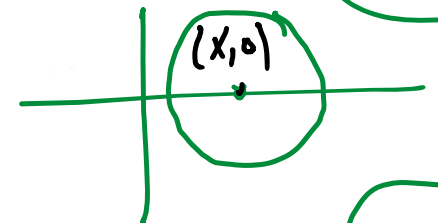
The equation is free of the absolute term *i.e.* $(C = 0)$



2 Equation of the circle whose centre lies on x-axis :

$$x^2 + y^2 + 2Lx + C = 0$$

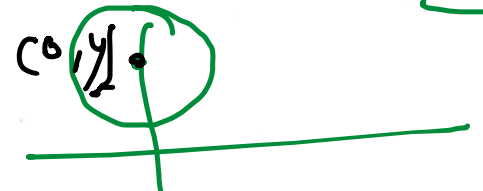
The equation is free of the term containing y *i.e.* $(K = 0)$



3 Equation of the circle whose centre lies on y-axis :

$$x^2 + y^2 + 2Ky + C = 0$$

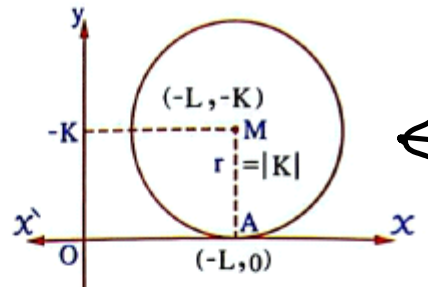
The equation is free of the term containing x *i.e.* $(L = 0)$



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4 Equation of the circle touching X-axis :

If the circle whose centre $(-L, -K)$ touches X-axis then :
 the point of tangency A is : $(-L, 0)$ and $r = |K|$
 $\therefore C = L^2 + K^2 - r^2 = L^2 + K^2 - K^2 = L^2$
 Then the equation of the circle becomes :

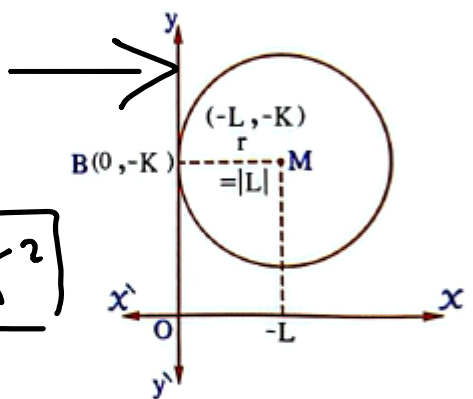


$C = L^2$

$x^2 + y^2 + 2Lx + 2Ky + L^2 = 0$

5 Equation of the circle touching y-axis :

If the circle whose centre $(-L, -K)$ touches y-axis , then the point of tangency B is $(0, -K)$ and $r = |L|$
 $\therefore C = L^2 + K^2 - r^2 = L^2 + K^2 - L^2 = K^2$
 , then the equation of the circle becomes :

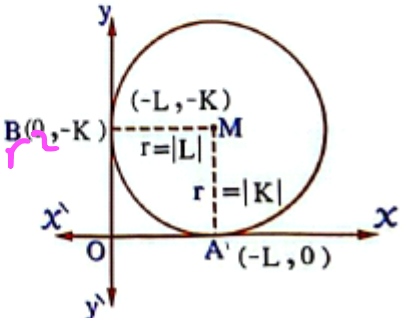


$C = K^2$

$x^2 + y^2 + 2Lx + 2Ky + K^2 = 0$

6 Equation of the circle touching the two coordinates :

If the circle whose centre is $(-L, -K)$ touches the two coordinates axes then $r = |L| = |K|$
 $\therefore C = L^2 + K^2 - r^2 = r^2 + r^2 - r^2 = r^2$
 $\therefore C = L^2 = K^2 = r^2$
 and the equation of the circle becomes :



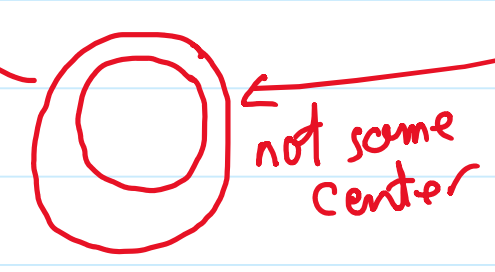
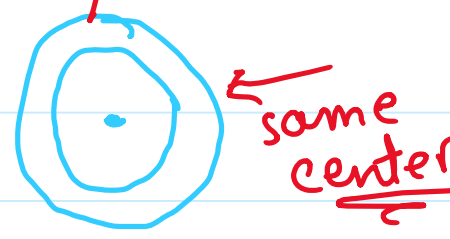
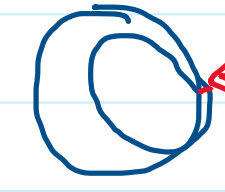
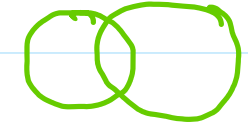
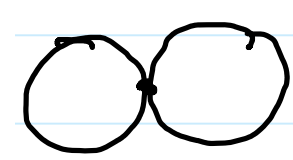
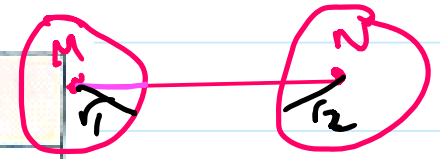
$C = L^2 = K^2 = r^2$

$x^2 + y^2 + 2Lx + 2Ky + C = 0$
 where $|L| = |K| = r, C = L^2 = K^2 = r^2$

$MN = \checkmark$
 $r_1 + r_2 = \checkmark$

(2) If M, N are two circles of radii r_1, r_2 respectively (where $r_1 > r_2$)

If the two circles M and N	Then
(1) Distant	$MN > r_1 + r_2$ ←
(2) Touching externally	$MN = r_1 + r_2$
(3) Intersecting	$r_1 - r_2 < MN < r_1 + r_2$ ←
(4) Touching internally	$MN = r_1 - r_2$ ←
(5) One inside the other	$MN < r_1 - r_2$ ←
(6) Concentric	$MN = \text{zero}$ ←



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(3) The tangent to a circle is perpendicular to the radius drawn from the point of tangency.

(4) The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

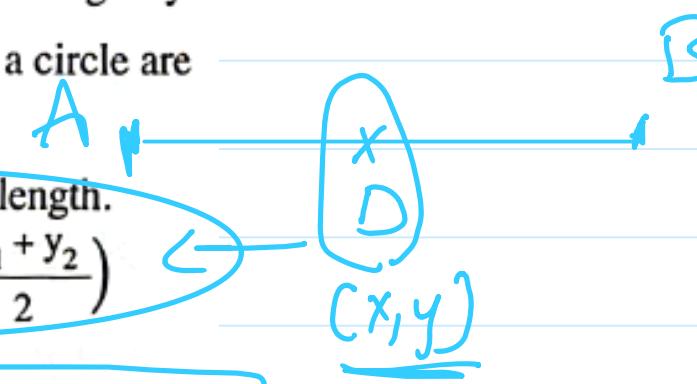
(5) The two tangent-segments drawn to a circle from a point outside it are equal in length.

(6) If $A = (x_1, y_1)$, $B = (x_2, y_2)$ then the midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

(7) The equation of the straight line passing through (x_1, y_1)

and its slope (m) is : $\frac{y - y_1}{x - x_1} = m$

$\frac{y_2 - y_1}{x_2 - x_1} = m$



(8) The length of the perpendicular from the point (x_1, y_1) on the straight line whose

equation : $a x + b y + C = 0$ equals $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

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Find the general form of the equation of the circle whose centre is $(-2, 3)$ and its radius length is 5 length units.

$$(x-d)^2 + (y-h)^2 = r^2$$

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$$x^2 + y^2 + \square x + \square y + \square = 0$$

$$(x-d)^2 + (y-h)^2 = r^2$$

$$(x+2)^2 + (y-3)^2 = 25$$

$$x^2 + y^2 + 2Lx + 2Ky + C = 0$$

$$x = \frac{-2L}{-2} + 2 = -L \quad \boxed{L=2}$$

$$y = \frac{-2K}{-2} + 3 = -K \quad \boxed{K=-3}$$

$$x^2 + 4x + 4 + y^2 + 6y + 9 - 25 = 0$$

$$x^2 + y^2 + 4x - 6y - 12 = 0$$

expand / $\sqrt{}$

95%

$$r^2 = L^2 + K^2 - C$$

$$5^2 = 4 + 9 - C \quad C = 13 - 25$$

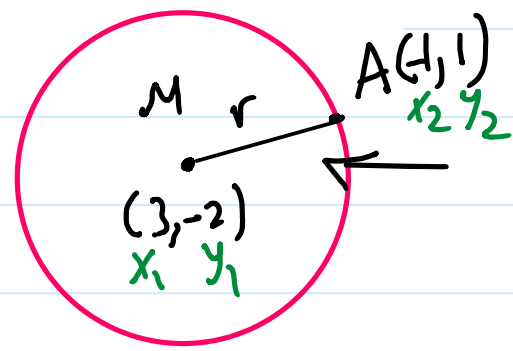
$$\boxed{C = -12}$$

$$x^2 + y^2 + 4x - 6y - 12 = 0$$

Find the equation of the circle whose centre $M = (3, -2)$ and passing through the point $A = (-1, 1)$

$$MA = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Distance



$$MA = \sqrt{(1 - (-2))^2 + (-1 - 3)^2}$$

$$= 5$$

$$L = -3 \quad K = 2$$

$$r^2 = (-3)^2 + 2^2 - C$$

$$x^2 + y^2 - 6x + 4y - 12 = 0$$

$$C = 9 + 4 - 25$$

$$\boxed{C = -12}$$

Find the equation of the circle whose centre (3, -4) and touches X-axis:

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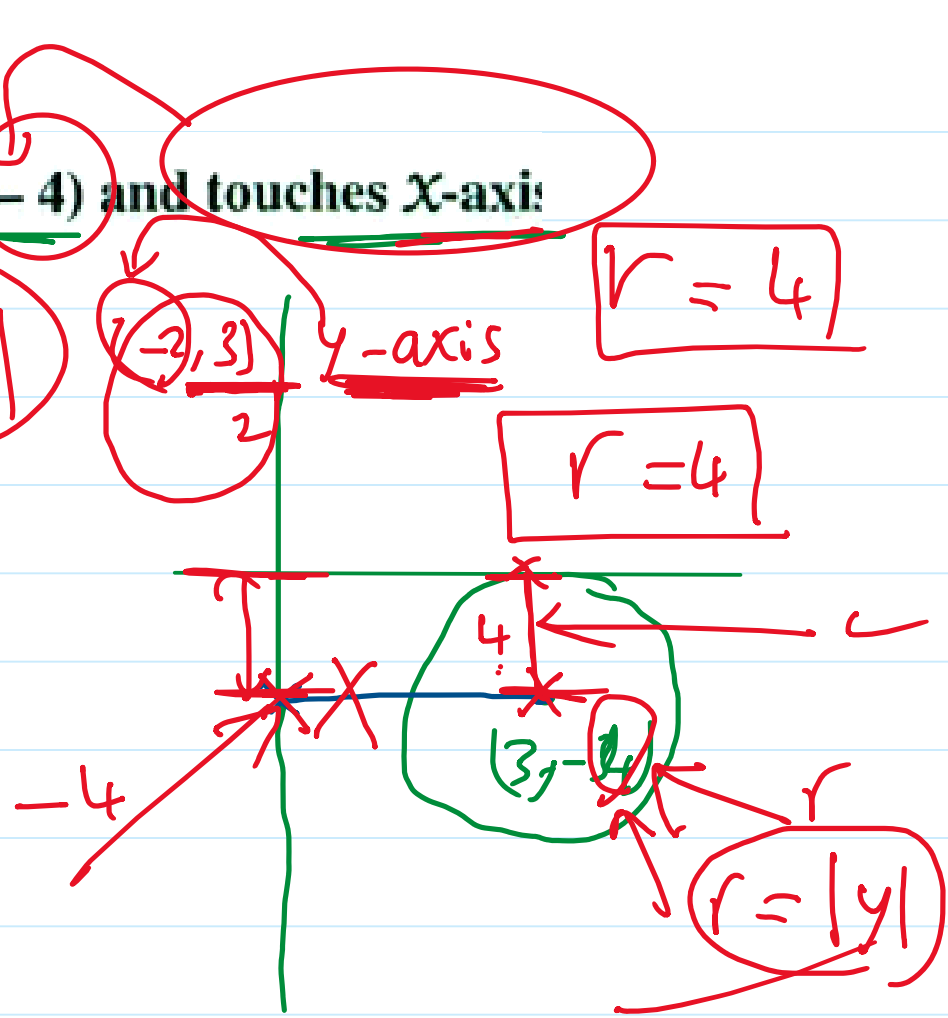
C = L²

r = |x|

r = 4

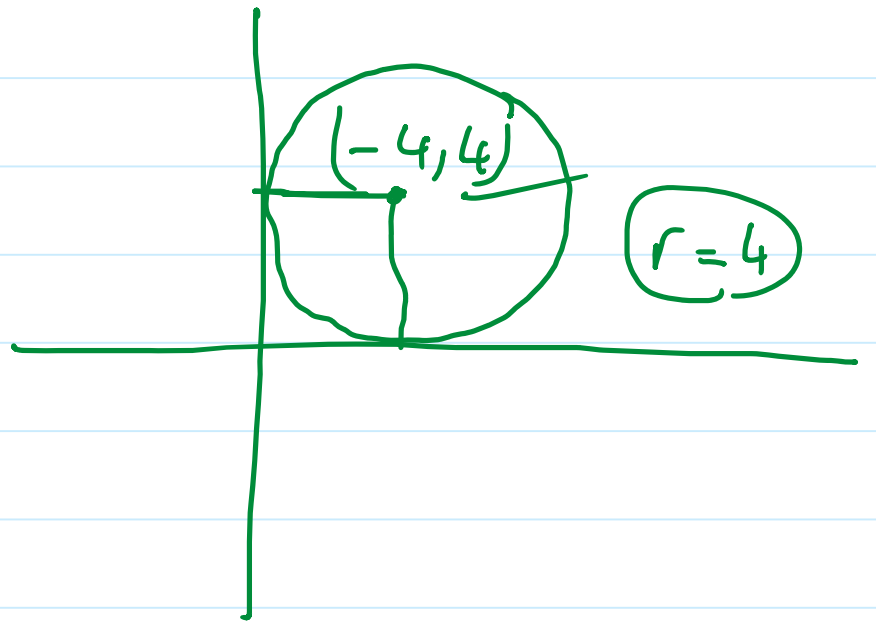
r = 4

x² + y² + 2Lx + 2Ky + C = 0



Find the equation of the circle which touches the two coordinate axes, and its centre is the point (-4, 4)

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- 15 Find equation of a circle which passes through the two points $A(1, 3)$, $B(2, -4)$ and its center lies on x-axis.

center = (x, y)

$$r = ?$$

$$r_1^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$$

$$= (y - 3)^2 + (x - 1)^2$$

$$r_1^2 = y^2 - 6y + 9 + x^2 - 2x + 1 \rightarrow (1)$$

$$r_2^2 = (y + 4)^2 + (x - 2)^2$$

$$r_2^2 = y^2 + 8y + 16 + x^2 - 4x + 4$$

$$8y - 4x + 20 = -6y - 2x + 10$$

$$\cancel{14y} - \frac{2x}{-2} = \frac{-10}{-2}$$

$$x = 5$$

