

Logarithmic Function_1

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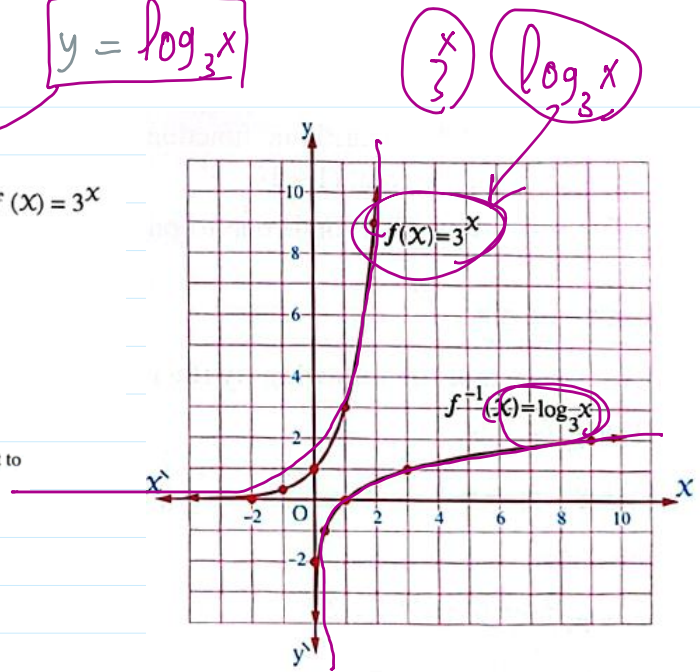
$y = 3^x$
 $\log_3 x = y$
 $y = \log_3 x$

We had studied before the graph of the exponential function $f : f(x) = 3^x$ i.e. $y = 3^x$, then we form the following table :

x	-2	-1	0	1	2
$f(x) = 3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

By replacing the two variables we get the inverse function $x = 3^y$ which is equivalent to the logarithmic form $y = \log_3 x$ i.e. $f^{-1}(x) = \log_3 x$

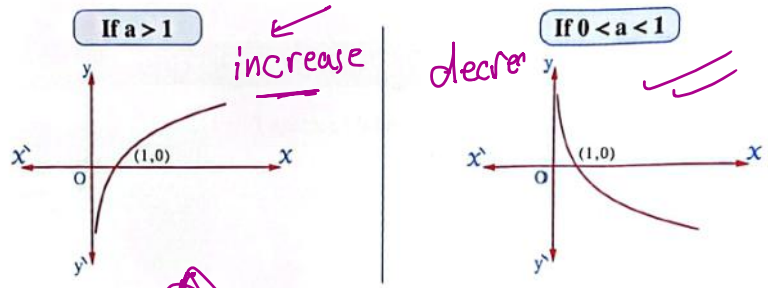
x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$f^{-1}(x) = \log_3 x$	-2	-1	0	1	2



The logarithmic function is the inverse function for the exponential function.

The graphical representation of the logarithmic function $f : f(x) = \log_a x$

The graph of the logarithmic function will be in the shape of one of the following figures according to the value of the base a :



$a \neq -ve$
 $\log_a x$
 $0 < a < 1$
 $a > 1$
 Domain $\rightarrow \mathbb{R}$
 Range $\rightarrow \mathbb{R}^+$
 exponent fun.
 3^x
 Error
 Zero

Some properties of the logarithmic function $f : f(x) = \log_a x$

- (1) The domain of the logarithmic function $= \mathbb{R}^+$
- (2) The range of the logarithmic function $= \mathbb{R}$
- (3) The logarithmic function is increasing when $a > 1$ and it is decreasing when $0 < a < 1$
- (4) All curves of the logarithmic functions for any positive base $\neq 1$ pass through the point $(1, 0)$
- (5) The logarithmic function is one-to-one i.e. If $\log_a x = \log_a y$, then $x = y$

$a \neq -ve$
 $\log 1 \rightarrow \text{Zero}$

pass through the point $(1, 0)$

(5) The logarithmic function is one-to-one i.e. If $\log_a x = \log_a y$, then $x = y$

$\log_a y \rightarrow x = y$

Express each of the following by the equivalent exponential form :

(1) $\log_2 64 = 6$

(3) $\log_3 \frac{1}{27} = -3$

(2) $\log_2 8\sqrt{2} = \frac{7}{2}$

(4) $\log 0.01 = -2$

$\log_2 64 \Rightarrow 6$
 $64 = 2^6$

$\log_a x = y$
 $x = a^y$

$\log 0.01 = -2$
 $0.01 = 10^{-2}$

Write the logarithmic form that is equivalent to each of the following exponential forms :

(1) $243 = (\sqrt{3})^{10}$

(3) $3^{\frac{5}{2}} = 9\sqrt{3}$

(2) $10^{-2} = 0.01$

(4) $c = a^x$ exp. \log

$\log_a c = x$
 (a) base

$\log_a c = x$

$3^{\frac{5}{2}} = 9\sqrt{3} \longrightarrow \text{logarithm.}$

$\log_3 9\sqrt{3} = \frac{5}{2}$
 $\log \square$
 $\log \square$

Find the value of each of :

(1) $\log_2 64$

(4) $\log_{\sqrt{3}} \frac{1}{27}$

(2) $\log_6 1$

(5) $\log 0.0001$

(3) $\log_4 4\sqrt{2}$

$\log_4 4\sqrt{2} = X \Rightarrow$

$$4\sqrt{2} = 4^X$$

$$2^{\downarrow 2} * 2^{\frac{1}{2}} = 2^{2X}$$

$$2^{2+\frac{1}{2}} = 2^{2X}$$

$$2 = 2$$

$\therefore 2X = \frac{5}{2} \Rightarrow X = \frac{5}{4}$

$\log_{10} 0.0001 = X$

(10)

$0.0001 = 10^X$

$10^{-4} = 10^X$

Find the value of X if :

(1) $\log_2 X = -4$

(3) $\log_{\frac{1}{2}} X = -3$

(2) $\log_9 81\sqrt{3} = X$

(4) $\log_x 8 = 6$

$\log_2 X = -4$

$X = 2^{-4}$

\log_x

$X^6 = 8$

$(X^2)^3 = 2^3$

$\sqrt{X^2} = \sqrt{2}$

$X = \pm\sqrt{2} \Rightarrow X$

$X = -\sqrt{2}$ refused

Find in \mathbb{R} the solution set of each of the following equations :

(1) $\log_x 7x = 2$

(3) $\log_2 \log_3 (x^2 - 7x + 21) = 1$

(2) $\log_2 \left(x^2 + \frac{3}{4}x\right) = -2$

(4) $(\log_2 x)^2 - 3 \log_2 x = 4$

$\log_2 \left(x^2 + \frac{3}{4}x\right) = -2$

$x^2 + \frac{3}{4}x = 2$

$x^2 + \frac{3}{4}x = \frac{1}{4}$

$4x^2 + 3x - 1 = 0$

Handwritten notes: "2nd sec" with an arrow pointing to the exponent -2; "2 prep" with an arrow pointing to the coefficient 4; "x^2" with an arrow pointing to the x^2 term in the first equation; "x^2" with an arrow pointing to the x^2 term in the second equation; "x^2" with an arrow pointing to the x^2 term in the third equation; "x^2" with an arrow pointing to the x^2 term in the fourth equation.