

Some properties of logarithms_1

Friday, December 12, 2025 4:41 PM

$$\log_5 5 = 1$$

$$\log_{10} 10 = 1$$

1st Property

• If $a \in \mathbb{R}^+ - \{1\}$, then $\log_a a = 1$

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2nd Property

• If $a \in \mathbb{R}^+ - \{1\}$, then $\log_a 1 = 0$

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3rd Property Multiplication property

• If $x, y \in \mathbb{R}^+$, $a \in \mathbb{R}^+ - \{1\}$

, then $\log_a xy = \log_a x + \log_a y$

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$$\log 3 \times 5 = \log 3 + \log 5$$

$$\log \left(\frac{3}{5}\right) = \log 3 - \log 5$$

$$\log 13 - \log 15 = \log \frac{13}{15}$$

$$\log 5 - \log 3 \rightarrow X$$

$$\log 5 + \log 4 =$$

$$\log_3 5 \times 4 = \log_3 20$$

$$\log_3 2 = 5 \log_3 2$$

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5th Property The power property

• If $x \in \mathbb{R}^+$, $a \in \mathbb{R}^+ - \{1\}$, $n \in \mathbb{R}$, then $\log_a x^n = n \log_a x$

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$$\log_6 4 + \log_6 3 = \log_6 4 + \frac{\log_6 3}{\log_6 4}$$

$$\log_4 3 = \frac{\log_3 3}{\log_4 4}$$

6th Property Base changing property

• If $x \in \mathbb{R}^+$, $y \in \mathbb{R}^+ - \{1\}$, $a \in \mathbb{R}^+ - \{1\}$, then $\log_y x = \frac{\log_a x}{\log_a y}$

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7th Property The multiplicative inverse property

• If $x, y \in \mathbb{R}^+ - \{1\}$, then $\log_y x = \frac{1}{\log_x y}$

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$$\log_4 5 = \frac{1}{\log_5 4}$$

Some properties of logarithms_2

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Without using the calculator, find the value of each of the following:

- (1) $\log_3 15 + \log_3 6 - \log_3 10$
- (2) $\log_5 100 - 3 \log_5 2 - \log_5 18 + \log_5 36$
- (3) $\log_5 \frac{3}{5} + 2 \log_5 \frac{15}{2} - \log_5 \frac{5}{36} + \log_5 \frac{5}{243}$
- (4) $\log_2 7 \times \log_7 11 \times \log_{11} 9 \times \log_3 2$
- (5) $\frac{\log_2 243 - \log_3 32}{\log_2 27 - \log_3 8}$

$$\log_3 15 + \log_3 6 - \log_3 10 = 2$$

$$\log_3 90 - \log_3 10 = \log_3 \frac{90}{10}$$

$$= \log_3 9 = \log_3 3^2 = 2 \log_3 3 = 2$$

$$\log_5 \frac{3}{5} + 2 \log_5 \frac{15}{2} - \log_5 \frac{5}{36} + \log_5 \frac{5}{243} = 2$$

$$\log_5 \left(\frac{3}{5} \times \left(\frac{15}{2}\right)^2 \times \frac{5}{243} \right) = \log_5 5 = 1$$