

# Some properties of logarithms\_3

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Without using the calculator, prove that :

(1)  $3 \log 5 + 2 \log 6 - \log 9 + \log 0.2 = \log_5 25$

(2)  $\frac{\log 30 - \log 6 + \log 5}{\log 12 - \log 3 + \log 25} = 1 - \log 2$

$\frac{5 \rightarrow 9 \rightarrow 3}{\dots} =$   
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①  
L.H.S =  $3 \log 5 + 2 \log 6 - \log 9 + \log 0.2$   
 $= \log 5^3 + \log 6^2 - \log 9 + \log 0.2$   
 $= \log \frac{5^3 \times 6^2 \times 0.2}{9} = \log \frac{100}{10} = \log 10^2$   
 $= 2 \log 10 = 2 \times 1 = 2$

R.H.S =  $\log_5 25 = \log_5 5^2 = 2 \log_5 5 = 2$   
 $\therefore \text{L.H.S} = \text{R.H.S}$

# Some properties of logarithms\_4

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Find the value of each of the following in the simplest form :

(1)  $\log_2 \sqrt[7]{32}$

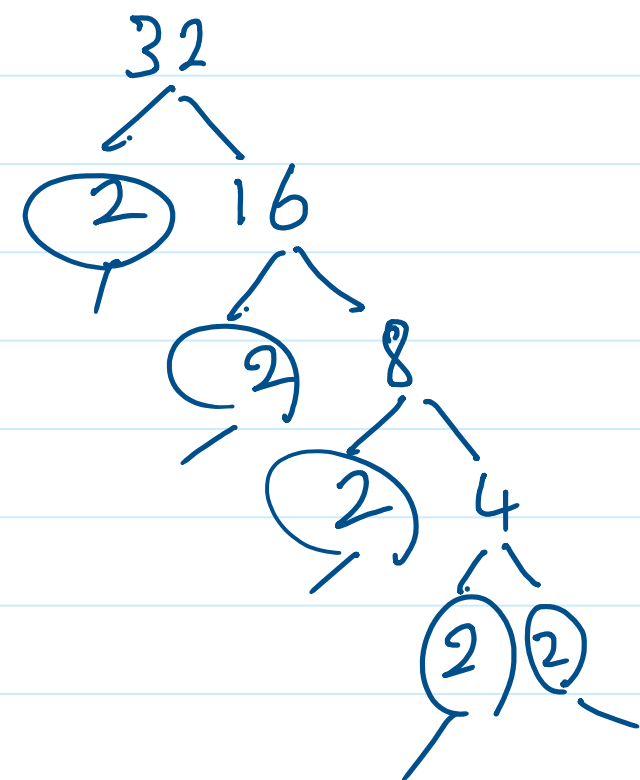


(2)  $\frac{1}{\log_x xyz} + \frac{1}{\log_y xyz} + \frac{1}{\log_z xyz}$

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$$\log_2 2^{5/7} = \frac{5}{7} (\log_2 2) = \frac{5}{7}$$

$$\begin{aligned} \textcircled{2} \log_{xyz} x + \log_{xyz} y + \log_{xyz} z \\ = \log_{xyz} xyz = 1 \end{aligned}$$



# Some properties of logarithms\_5

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Using the calculator, find the value of  $x$  to the nearest 2 decimal digits in each of the following:

(1)  $5^x = 17$

(3)  $5^{x+1} = 2^{4x-3}$

(5)  $3^{2x} - 14 \times 3^x + 45 = 0$

(2)  $2^{x-1} = 7$

(4)  $5^{x-2} = 3 \times 4^{x+1}$

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$\log 5^x = \log 17$   $x = ??$

$\frac{x \log 5}{\log 5} = \frac{\log 17}{\log 5} \Rightarrow x = \frac{\log 17}{\log 5} \approx 1.76$

$4 \log 5^{x-2} = \log 3 \times 4^{x+1}$

~~$x-2 \log 5 = \log 3 + \log 4^{x+1}$~~   
 ~~$x-2 \log 5 = \log 3 + x+1 \log 4$~~   
 ~~$x-2 \log 5 - x+1 \log 4 = \log 3$~~

$\log 5^{x-2} = \log 3 \times 4^{x-2}$

$\log 5 \cdot 5 = \log 3 \times 4 \times 4^1$

$\log \frac{5^x}{5^2}$

$\log 5^x - \log 5^2 = \log 3 + \log 4 + \log 4$

$x \log 5 - x \log 4 = \log 3 + \log 4 - \log 25$

$\frac{x(\log 5 - \log 4)}{\log 5 - \log 4} = \frac{\log 3 + \log 4 - \log 25}{\log 5 - \log 4}$

$x = -3.28$

# Some properties of logarithms\_6

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## Important remark at solving logarithmic equation

If  $x \in \mathbb{R}^*$  and  $m$  is an even number  $\neq 0$  and  $a \in \mathbb{R}^+ - \{1\}$

, then  $\log_a x^m = m \log_a |x|$

For example :  $\log_5 x^4 = 4 \log_5 |x|$

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Find in  $\mathbb{R}$  the S.S. of each of the following equations :

(1)  $2 \log x - \log(x+2) = 0$

(2)  $\log x^2 = \log 4 + \log 9$

(3)  $\log_2 x + \log_2(x-2) = 3$

(4)  $\log x + \log(x+2) = \log(x+6)$

(5)  $\log \sqrt[3]{2x-1} + \log \sqrt[3]{x-2} = \log 30 - 1$

(6)  $\frac{\log 49 - (\log 7)^2}{\log 0.07} = \log x$

$$2x - x^2 = x(2-x)$$

$$2 \log 7 - (\log 7)^2$$

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$$\boxed{6} \quad \frac{\log 49 - (\log 7)^2}{\log 0.07} = \log x$$

$$\rightarrow \frac{2 \log 7 - (\log 7)^2}{\log \frac{7}{100}} = \log x$$

$$\frac{\log 7 (2 - \log 7)}{\log 7 - \log 100} = \log x$$

$$\log 7 - \log 100 \rightarrow \log 10^2 = 2 \log_{10} 10 = 2$$

$$\frac{\log 7 (2 - \log 7)}{(\log 7 - 2)} = \log x$$

$$\frac{-\log 7 (\log 7 - 2)}{\log 7 - 2} = \log x$$

$$\rightarrow \log 7 = \log x$$

$$\log 7^{-1} = \log x$$

$$\boxed{x = \frac{1}{7}}$$