

Some properties of logarithms_7

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Let

Let $y = \log_4 x$

$$\log_4 x + \frac{1}{\log_4 x} = 2$$

$$y + \frac{1}{y} = 2 \quad (*y)$$

$$y^2 + 1 = 2y$$

$$y^2 - 2y + 1 = 0$$

$$(y-1)^2 = 0$$

$$y = 1$$

$$\log_4 x = 1$$

$$x = 4^1$$

$$x = 4$$

Find in \mathbb{R} the S.S. of each of the following equations :

(1) $\log_3 (x^2 - 3x + 2) - \log_3 (x - 2) = \log_7 49$

(2) $x^{\log x} = 10$

(3) $\log_2 x = \log_4 25$

(4) $\log_3 x \times \log_9 x = 2$

(5) $\log_4 x + \log_x 4 = 2$

① R.H.S = $2 \log_7 7 = 2$

L.H.S = $\log_3 (x-2)(x-1) - \log_3 (x-2)$
 $= \cancel{\log_3 (x-2)} + \log_3 (x-1) - \cancel{\log_3 (x-2)}$

$\therefore \log_3 (x-1) = 2$

$x-1 = 3^2$

$x = 10$

$\log_3 (x^2 - 3x + 2) - \log_3 (x - 2) = 2 \log_7 7$

$\log_3 \left(\frac{x^2 - 3x + 2}{x - 2} \right) = 2$

$\frac{x^2 - 3x + 2}{x - 2} = 3^2$

$x^2 - 3x + 2 = 9x - 18$

$x^2 - 3x - 9x + 2 + 18 = 0$

$x^2 - 12x + 20 = 0$

$(x - 10)(x - 2) = 0$

$x = 10$ $x = 2$ → refused

$\log x \log x = 10$

$\log x = 1$

$\log x \log x = 1$
 $\sqrt{(\log x)^2} = \sqrt{1}$

$\log x = 1$
 $x = 10$

$\log x = -1$
 $x = 0.1$

$$\sqrt{(\log x)^2} = \sqrt{1}$$

$$\boxed{\log x = \pm 1}$$

$$|x = 10| \longleftrightarrow |x = 0.1|$$

$$S.S = \{0.1, 10\}$$

$$\frac{\log x}{\log 9}$$

$$\log_3 x \neq \log_9 x = 2$$

$$\log_3 x$$

$$\frac{\log_9 x}{\log_9 9}$$

2nd
sec

$$= 2 (\log_3 x)^2 \times \frac{1}{2} = 2$$

$$\sqrt{(\log_3 x)^2} = \sqrt{4}$$

$$\log_3 x = \pm 2$$

$$x = 3^2$$

$$x = 3^{-2}$$

$$\boxed{x = 9}$$

$$\boxed{x = \frac{1}{9}}$$

$$\log_2 5 * \log_5 2 = \log_2 5 * \frac{1}{\log_2 5} = 1$$

$$\log_3 81 * \log_9 3 = \log_3 3^4 * \frac{1}{\log_3 3} = 4 * 1 = 4$$

$$= \frac{4 \log_3 3}{2 \log_3 3} = \frac{4}{2} = 2$$

(15) $\log_5 \frac{1 - \log_3 2}{\log_3 \left(\frac{3}{2}\right)}$

$$= \log_5 \frac{1 - \log_3 2}{5[\log_3(3) - \log_3 2]}$$

$$= \log_5 \frac{1 - \log_3 2}{5(1 - \log_3 2)}$$

$$= \log_5 \frac{1}{5} = \log_5 5^{-1} = -1$$

(2) $\log_2 \left(\frac{3}{11}\right) + \log_2 \left(\frac{297}{98}\right) - (2 \log_2 2 * \log_2 \frac{9}{7}) = -1$

3	297
3	99
3	33
3	11
1	1

L.H.S =

$$\log_2 3 - \log_2 11 + \log_2 \frac{3^3 * 11}{7^2 * 2} - (2 * (\log_2 3 - \log_2 7))$$

$$= \log_2 3 - \log_2 11 + \log_2 3^3 + \log_2 11 - \log_2 7^2 - \log_2 2 - 4 \log_2 3 + 2 \log_2 7$$

$$= -1 = \text{R.H.S}$$

(19) $3^{\log_4 (x + 1.25)} = \frac{1}{3}$

$$\therefore \log_4 (x + 1.25) = -1$$

$$x + 1.25 = 4^{-1}$$

$$x = \frac{1}{4} - 1.25$$